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Regulars

Mathematical mysteries: Getting the most out of life – Part 1

by Mark Wainwright



There are many sorts of games played in a "bunco booth", where a trickster or sleight-of-hand expert tries to relieve you of your money by getting you to place bets – on which cup the ball is under, for instance, or where the queen of spades is. Lots of these games can be analysed using probability theory, and it soon becomes obvious that the games are tipped heavily in favour of the trickster! The punter is well advised to steer clear. If, as in the game I'll describe here, the probabilities don't seem to favour the trickster, steer clear anyway. He's probably bending the odds with some classy sleight of hand.

The idea of this game is that I write two cheques for different amounts of money, put each cheque in an envelope, and offer you the two envelopes. You are allowed to choose one envelope and look at its contents. Then you have to decide which one to keep – the one you've opened, or the other one, which you are not allowed to look inside before you make your decision.

What can you lose? This game sounds like free money. Unfortunately, the cheques aren't real; they'll bounce if you try to cash them. All the same, as a point of pride, you'd like to end up with the larger of the two amounts. In fact, let's make a bet: I bet you'll end up with the smaller amount. Do you take the bet?

By choosing at random, you will succeed 50% of the time. This is a fair bet, like betting on the toss of a coin. If you could do even slightly better than this, you would make a profit in the long run if we played the game many times. However, it seems obvious that, unless you know something about what numbers I am likely to choose, you cannot succeed more often than 50% of the time.

It's better than you think

However, the surprising fact is that it *is* possible for you to use a strategy that succeeds more often than 50% of the time. What's more, it doesn't depend on any assumptions about what numbers I will choose. In fact, even if I know your strategy and I try to choose numbers that will break it, I won't be able to – you will still succeed with probability that is strictly greater than 50%.

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So how does this strategy work? Before I tell you, you might like to give it some thought. This is quite a difficult problem, so if you don't find an answer at first, sleep on it for a day or two. You might like to assume that the cheques are in some ordinary currency, like sterling, so they are both for (positive) whole numbers of pence. Actually, even if I could write cheques for any real numbers, like pi or the square root of two, you would still be able to prevail. But it is easier to think about if the amounts are restricted to positive whole numbers.

Here's a last hint: the bigger the cheque you find in the first envelope, the more you will be inclined to keep it rather than swap it for some other, unknown amount. So you should try to arrange that the larger the cheque you see, the more likely you are to keep it.

When you find an answer, or give up, carry on and read the second part of the article.



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