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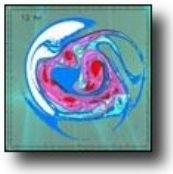
Features



The Mathematics of Fibonacci's Sequence

by Keith Moffatt

The Fibonacci sequence is defined by the property that each number in the sequence is the sum of the previous two numbers; to get started, the first two numbers must be specified, and these are usually taken to be 1 and 1. In mathematical notation, if the sequence is written $(x_0, x_1, x_2, ...)$ then the defining relationship is

$$x_n = x_{n-1} + x_{n-2} \qquad (n = 2, 3, 4...)$$

with starting conditions $x_0 = 1, x_1 = 1$. On dividing both sides of (1) by x_{n-1}, we obtain $1/R_n = 1 + R_{n-1}$ where $R_n = x_{n-1}/x_n$, the ratio of successive terms.

As $n \to \infty, R_n \to R$ where $1/R = 1 + R$, or $R^2 + R - 1 = 0.$ This quadratic equation has two roots; the one we need here is obviously between zero and one; it is

$$R = \frac{\sqrt{5} - 1}{2} = 0.61803...,$$

the number known as the Golden Ratio.

The number R has some remarkable properties; for example, it is expressible as a "continued fraction":

$$R = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

In the theory of chaotic dynamical systems, R is recognised as "the most irrational number" between 0 and 1!

The spiral curve shown in the poster is a logarithmic spiral, a curve whose equation in polar coordinates is $r = ke^{a\theta}$ where *k* and *a* are constants. The spiral patterns evident in the sunflower are of this form, and the

numbers of spirals going in opposite senses are the conscutive Fibonaci numbers 34 and 55. The underlying reason for this may be found in many texts; see for example Conway JH and Guy RK **The Book of Numbers,** Springer−Verlag (1996), chapter 4.

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Keith Moffatt is a fellow of the Royal Society and Director of the Isaac Newton Institute for Mathematical Sciences, a national and international visitor research institute at the University of Cambridge.

His own research interests lie in the field of fluid dynamics.