



© 1997–2004, Millennium Mathematics Project, University of Cambridge.

Permission is granted to print and copy this page on paper for non-commercial use. For other uses, including electronic redistribution, please contact us.

May 1997

Regulars

Mathematical mysteries: the Goldbach conjecture



Prime numbers provide a rich source of speculative mathematical ideas. Some of the mystical atmosphere that surrounds them can be traced back to Pythagoras and his followers who formed secret brotherhoods in Greece, during the 5th Century BC. The Pythagoreans believed that numbers had spiritual properties. The discovery that some numbers such as the square root of 2 cannot be expressed exactly as the ratio of two whole numbers was so shocking to Pythagoras and his followers that they hushed up the proof!

Today, prime numbers are fascinating but they are also of commercial importance, since the best commercial and military ciphers depend on their properties. (See "[Discovering new primes](#)" in Issue 1 – it is yet to be proved that there are infinitely many Mersenne primes.)

Here is another unproved conjecture about prime numbers. It is called the Goldbach conjecture and may be stated as follows:

Every even number greater than 4 can be written as the sum of two odd prime numbers. } (G)

For example:

- $8 = 3 + 5$. Both 3 and 5 are prime numbers.
- $20 = 13 + 7 = 17 + 3$.
- $42 = 23 + 19 = 29 + 13 = 31 + 11 = 37 + 5$.

Notice that there can be more than one Goldbach pair. The conjecture says only that there is at least one, and has nothing to say about whether there may be more.

You can explore the Goldbach conjecture yourself with this [Goldbach calculator](#). Simply enter an even integer, n , greater than 4 and the calculator will find all the Goldbach pairs.

Historical Note

Christian Goldbach (1690–1764) was a Prussian amateur mathematician and historian who lived in St Petersburg and Moscow. He made his conjecture in a letter to Leonhard Euler, who at first treated the letter with some disdain, regarding the result as trivial. Goldbach's conjecture, however, remains unproved to this

Mathematical mysteries: the Goldbach conjecture

day.

Further reading

For an entertaining and revealing introduction to this problem, see Douglas R Hofstadter's book "Gödel, Escher, Bach" (Penguin Books 1979, ISBN 0 14 00 5579 7), especially the section "Aria with Diverse Variations" following chapter XII.



Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.