



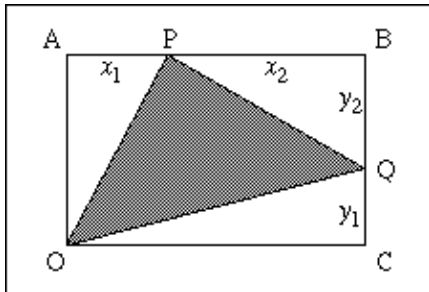
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Regulars

## Rectangle triangle problem



If we label the distances AP, PB, CQ and QB as shown above then we can write three equations for the areas of the triangles as follows:

$$\text{Area of OAP} = \frac{1}{2} x_1 (y_1 + y_2) \quad (1)$$

$$\text{Area of PBQ} = \frac{1}{2} x_2 y_2 \quad (2)$$

$$\text{Area of OCQ} = \frac{1}{2} (x_1 + x_2) y_1 \quad (3)$$

From (1) and (3):

$$x_1 y_1 + x_1 y_2 = x_1 y_1 + x_2 y_1$$

$$x_1 y_2 = x_2 y_1$$

$$\frac{x_2}{x_1} = \frac{y_2}{y_1}$$

Notice that the two ratios are the same! If we call this ratio  $r$  then we can calculate a polynomial for  $r$  as follows:

## Rectangle triangle problem

From (1) and (2):

$$x_1 y_1 + x_1 y_2 = x_2 y_2$$

$$y_1 + y_2 = \frac{x_2}{x_1} y_2 = r y_2$$

$$1 + \frac{y_2}{y_1} = r \frac{y_2}{y_1}$$

$$1 + r = r^2$$

$$r^2 - r - 1 = 0$$

Taking the positive root gives us the golden ratio:

$$r = \frac{1}{2}(1 + \sqrt{5}) \approx 1.618$$



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