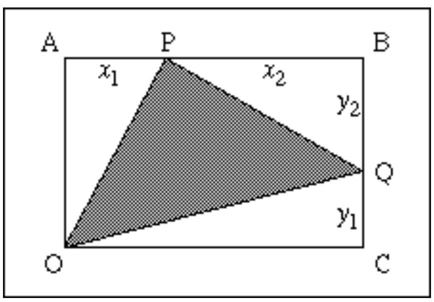
September 1997
Regulars

Rectangle triangle problem



If we label the distances AP, PB, CQ and QB as shown above then we can write three equations for the areas of the triangles as follows:

$$\text{Area of OAP} = \frac{1}{2}x_1(y_1 + y_2) \qquad (1)$$

$$\text{Area of PBQ} = \frac{1}{2}x_2 y_2 \qquad (2)$$

$$\text{Area of OCQ} = \frac{1}{2}(x_1 + x_2)y_1 \qquad (3)$$

From (1) and (3):

$$x_1 y_1 + x_1 y_2 = x_1 y_1 + x_2 y_1$$

$$x_1 y_2 = x_2 y_1$$

$$\frac{x_2}{x_1} = \frac{y_2}{y_1}$$

Notice that the two ratios are the same! If we call this ratio *r* then we can calculate a polynomial for *r* as follows:

From (1) and (2):

$$x_1 y_1 + x_1 y_2 = x_2 y_2$$

$$y_1 + y_2 = \frac{x_2}{x_1} y_2 = r y_2$$

$$1 + \frac{y_2}{y_1} = r \frac{y_2}{y_1}$$

$$1 + r = r^2$$

$$r^2 - r - 1 = 0$$

Taking the positive root gives us the golden ratio:

$$r = \frac{1}{2}(1 + \sqrt{5}) \approx 1.618$$

Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.