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Regulars

Outer space: Racing certainties

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A few months ago I saw a TV crime drama that involved a plan to defraud bookmakers by nobbling the favourite for a race. The drama centred around other events and the basis for the betting fraud was never explained. What might have been going on?

Suppose that you have a race where there are published odds on the competitors of a_1 to 1, a_2 to 1, a_3 to 1, and so on for any number, N, of runners. For example, if the odds are 5 to 4 then we express that as an a_i of 5/4 to 1. Now if we lay bets on all of the N runners in proportion to the odds so that we bet a fraction $1/(a_i+1)$ of the total stake money on the runner with odds of a_i to 1 then we will always show a profit so long as the odds satisfy the inequality

$$Q = \sum_{i=1}^{N} \frac{1}{a_i + 1} < 1,$$

and if Q is less than 1 then our winnings will be at least equal to

$$W = \left[\frac{1}{Q} - 1\right] \times \text{our total stake.}$$

Letâ€™s look at some examples. Suppose there are four runners and the odds for each are 6 to 1, 7 to 2, 2 to 1 and 8 to 1. Then we have $a_1 = 6$, $a_2 = 7/2$, $a_3 = 2$ and $a_4 = 8$ and

$$Q = \frac{1}{7} + \frac{2}{9} + \frac{1}{3} + \frac{1}{9} = \frac{51}{63} < 1$$

and so by betting our stake money with 1/7 on runner 1, 2/9 on runner 2, 1/3 on runner 3, and 1/9 on runner 4 we will win at least 51/63 of the money we staked (and of course we get our stake money back as well).

However, suppose that in the next race the odds on the four runners are 3 to 1, 7 to 1, 3 to 2 and 1 to 1 (ie evens). Now we see that we have

$$Q = \frac{1}{4} + \frac{1}{8} + \frac{2}{5} + \frac{1}{2} = \frac{51}{40} > 1$$

and there is no way that we can guarantee a positive return. Generally, we can see that if there is a large field of runners (N is big) there is is likely to be a better chance of Q being greater than 1. But large N doesnâ€™t guarantee $Q > 1$. Pick each of the odds by the formula $a_i = i(i+2) - 1$ and you can get $Q = 3/4$ and a healthy 30% return even when N is infinite!

But let's return to the TV programme. How is the situation changed if we know ahead of the race that the favourite in our $Q > 1$ example will not be a contender because he has been doped?
