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Features



Exploring the Enigma

by Claire Ellis



As long ago as the Ancient Greeks, warring armies have encrypted their communications in an attempt to keep their battle plans a secret from their enemies. However, just as one side invented an ingenious new way to encipher its messages, so would its enemies discover a clever way of cracking that code. The result has been that codes and ciphers have become more and more complex and increasingly difficult to crack over time, as, throughout history, an intellectual battle has raged between code makers and code breakers.

The battle of wits was never keener than during the Second World War, when the Germans used the famous Enigma machine – which they believed uncrackable – to encode messages, and the Allies worked at Bletchley Park to decipher the code.

The birth of an enigma



German soldiers using an Enigma machine during the second world war

Up till the Second World War, the most advanced forms of encryption involved simple paper and pencil techniques. But security blunders on both sides during the First World War highlighted a need for a higher level of secrecy, with more advanced methods of enciphering messages. Both the Allies and the Axis countries were looking for a new way to encrypt messages – a way that would result in complete security. (For more information, have a look at our [explanation of the basic terminology of codes and ciphers](#).)

In 1915 two Dutch Naval officers had invented a machine to encrypt messages. This encryption tool became one of the most notorious of all time: the Enigma cipher machine. Arthur Scherbius, a German businessman, patented the Enigma in 1918 and began selling it commercially to banks and businesses.

The Enigma machine's place in history was secured in 1924 when the German armed forces began using a specially adapted military version to encrypt their communications. They continued to rely on the machine throughout the Second World War, believing it to be absolutely unbreakable.

How the Enigma machine worked

Exploring the Enigma



A diagrammatic representation of an Enigma machine

When a plaintext letter was typed on the keyboard, an electric current would pass through the different scrambling elements of the machine and light up a ciphertext letter on the "lamp board". What made the Enigma machine so special was the fact that every time a letter was pressed, the movable parts of the machine would change position so that the next time the same letter was pressed, it would most likely be enciphered as something different. This meant that it wasn't possible to use traditional methods to try and crack the notorious cipher.

To make things even more difficult, different parts of the machine could be set up in different ways, with each setting producing a unique stream of enciphered letters. Unless you knew the exact settings of the machine, you couldn't decipher the messages.

How many ways are there to set up an Enigma Machine?

Choosing Rotors



An Enigma machine rotor. Copyright Simon Singh

Army issue Enigma machines had three revolving "wheels" or "rotors" that could be taken out and changed about. The first task for an Enigma operator would be to decide which rotor went in which position. There were five rotors to choose from and they could be inserted into three positions on the Enigma machine.

- **Question 1:** How many possible ways are there of positioning 5 rotors in 3 slots in the Enigma?
([Check your answer.](#))

The rotor starting positions

- **Question 2:** Once you have chosen the order of the rotors, how many possible ways can you set the starting positions of the rotors?
([Check your answer.](#))

The ring settings

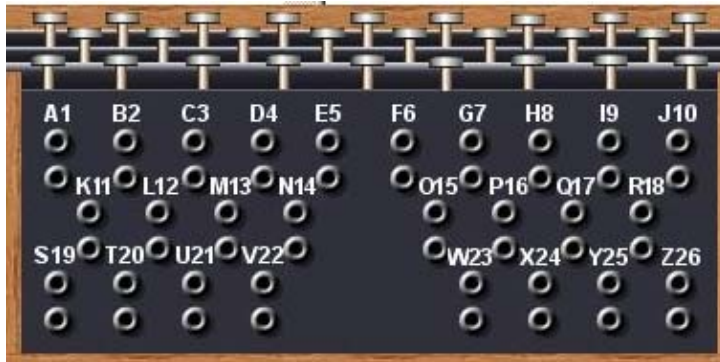
Every time a letter was pressed on the keyboard, the rotor on the far right would move around one place. Once it had completed a full revolution (ie moved forward 26 places), it would kick the middle rotor forward one position. When it had completed another revolution, it would again kick the middle rotor forward one position. When the middle rotor had completed a full revolution, it would kick the left-hand rotor forward.

The point at which the right hand rotor kicked the middle rotor forward and the point at which the middle rotor kicked the left hand rotor forward could be changed. This was called the "Ring Setting".

- **Question 3:** How many possible "Ring settings" were there on a 3-rotor army Enigma?
([Check your answer.](#))

The plugboard

Exploring the Enigma



A diagrammatic representation of the plugboard

On the front of the machine was another section called the "plugboard". The Enigma machine had several cables with a plug at each end that could be used to plug pairs of letters together. If A were plugged to B then, on typing the letter A, the electric current would follow the path that was normally associated with the letter B, and vice versa.

Enigma machines had 10 cables with which to link up pairs of letters.

- **Question 4:** How many ways are there to link up pairs of letters on the Enigma machine?

The answer is that there are approximately 150,000,000,000,000 – that is, 150 million million – possible combinations of 10 pairs of 26 letters on the plug board. The maths behind this calculation is complex, but a full explanation is given at <http://www.codesandciphers.co.uk/enigma/steckercount.htm>, a page from Tony Sale's website.

Therefore, the total number of possible ways in which a standard army-issue Enigma machine could be set up was:

$$60 \times 17,576 \times 676 \times 150,738,274,937,250,$$

which is approximately 158 million million million.

Deciphering Enigma

When the Enigma machine is used, the Enigma machine itself is the algorithm; the way in which it is set up is the key. Just as with any other type of cipher, as long as the recipient knows the key, the process of deciphering an Enigma encrypted message is incredibly simple. A German soldier receiving an enciphered message simply had to type the ciphertext letters into his own Enigma machine. If his machine was set up exactly in exactly the same way as the message sender's, then the plaintext letters would appear on the lamp board.

However, just as with any other type of cipher system, if you don't know the key it is very difficult to read the message – even if you know which system was used to encipher it.

The British had set up listening stations (called *Y Stations*) all over Britain, so that they could eavesdrop on the German military. Even though the Allies had managed to get hold of Enigma machines, in order to decrypt the messages they intercepted they needed to know the key. To make it as difficult as possible for the Allies to decipher messages, the Germans would change the key every day, resetting their Enigma machines at

midnight every night.

Agreeing on a key

Luftwaffen - Maschinen - Schlüssel Nr. 649 AIR FORCE OTHER CODE 649 No. 0011

Achtung! Schlüsselblätter dürfen nicht verändert in Feindeshand fallen. Bei Verlust sofortige Meldung an die Kommando- und Kontrollstellen.

Reihe	Tage	Mischtag Mischfunktion	Kriegsflagge Kriegsflagge	Strophensystem Strophensystem										Kombigruppen	
				an der Maschine an der Maschine											
				PLUS CORRECTION											
				1	2	3	4	5	6	7	8	9	10		
649	01	7	9	30	34	35	36	37	38	39	40	41	42	43	44
649	02	10	11	12	13	14	15	16	17	18	19	20	21	22	23
649	03	13	14	15	16	17	18	19	20	21	22	23	24	25	26
649	04	16	17	18	19	20	21	22	23	24	25	26	27	28	29
649	05	19	20	21	22	23	24	25	26	27	28	29	30	31	32
649	06	22	23	24	25	26	27	28	29	30	31	32	33	34	35
649	07	25	26	27	28	29	30	31	32	33	34	35	36	37	38
649	08	28	29	30	31	32	33	34	35	36	37	38	39	40	41
649	09	31	32	33	34	35	36	37	38	39	40	41	42	43	44
649	10	34	35	36	37	38	39	40	41	42	43	44	45	46	47
649	11	37	38	39	40	41	42	43	44	45	46	47	48	49	50
649	12	40	41	42	43	44	45	46	47	48	49	50	51	52	53
649	13	43	44	45	46	47	48	49	50	51	52	53	54	55	56
649	14	46	47	48	49	50	51	52	53	54	55	56	57	58	59
649	15	49	50	51	52	53	54	55	56	57	58	59	60	61	62
649	16	52	53	54	55	56	57	58	59	60	61	62	63	64	65
649	17	55	56	57	58	59	60	61	62	63	64	65	66	67	68
649	18	58	59	60	61	62	63	64	65	66	67	68	69	70	71
649	19	61	62	63	64	65	66	67	68	69	70	71	72	73	74
649	20	64	65	66	67	68	69	70	71	72	73	74	75	76	77
649	21	67	68	69	70	71	72	73	74	75	76	77	78	79	80
649	22	70	71	72	73	74	75	76	77	78	79	80	81	82	83
649	23	73	74	75	76	77	78	79	80	81	82	83	84	85	86
649	24	76	77	78	79	80	81	82	83	84	85	86	87	88	89
649	25	79	80	81	82	83	84	85	86	87	88	89	90	91	92
649	26	82	83	84	85	86	87	88	89	90	91	92	93	94	95
649	27	85	86	87	88	89	90	91	92	93	94	95	96	97	98
649	28	88	89	90	91	92	93	94	95	96	97	98	99	100	101
649	29	91	92	93	94	95	96	97	98	99	100	101	102	103	104
649	30	94	95	96	97	98	99	100	101	102	103	104	105	106	107
649	31	97	98	99	100	101	102	103	104	105	106	107	108	109	110
649	32	100	101	102	103	104	105	106	107	108	109	110	111	112	113
649	33	103	104	105	106	107	108	109	110	111	112	113	114	115	116
649	34	106	107	108	109	110	111	112	113	114	115	116	117	118	119
649	35	109	110	111	112	113	114	115	116	117	118	119	120	121	122
649	36	112	113	114	115	116	117	118	119	120	121	122	123	124	125
649	37	115	116	117	118	119	120	121	122	123	124	125	126	127	128
649	38	118	119	120	121	122	123	124	125	126	127	128	129	130	131
649	39	121	122	123	124	125	126	127	128	129	130	131	132	133	134
649	40	124	125	126	127	128	129	130	131	132	133	134	135	136	137
649	41	127	128	129	130	131	132	133	134	135	136	137	138	139	140
649	42	130	131	132	133	134	135	136	137	138	139	140	141	142	143
649	43	133	134	135	136	137	138	139	140	141	142	143	144	145	146
649	44	136	137	138	139	140	141	142	143	144	145	146	147	148	149
649	45	139	140	141	142	143	144	145	146	147	148	149	150	151	152
649	46	142	143	144	145	146	147	148	149	150	151	152	153	154	155
649	47	145	146	147	148	149	150	151	152	153	154	155	156	157	158
649	48	148	149	150	151	152	153	154	155	156	157	158	159	160	161
649	49	151	152	153	154	155	156	157	158	159	160	161	162	163	164
649	50	154	155	156	157	158	159	160	161	162	163	164	165	166	167
649	51	157	158	159	160	161	162	163	164	165	166	167	168	169	170
649	52	160	161	162	163	164	165	166	167	168	169	170	171	172	173
649	53	163	164	165	166	167	168	169	170	171	172	173	174	175	176
649	54	166	167	168	169	170	171	172	173	174	175	176	177	178	179
649	55	169	170	171	172	173	174	175	176	177	178	179	180	181	182
649	56	172	173	174	175	176	177	178	179	180	181	182	183	184	185
649	57	175	176	177	178	179	180	181	182	183	184	185	186	187	188
649	58	178	179	180	181	182	183	184	185	186	187	188	189	190	191
649	59	181	182	183	184	185	186	187	188	189	190	191	192	193	194
649	60	184	185	186	187	188	189	190	191	192	193	194	195	196	197
649	61	187	188	189	190	191	192	193	194	195	196	197	198	199	200
649	62	190	191	192	193	194	195	196	197	198	199	200	201	202	203
649	63	193	194	195	196	197	198	199	200	201	202	203	204	205	206
649	64	196	197	198	199	200	201	202	203	204	205	206	207	208	209
649	65	199	200	201	202	203	204	205	206	207	208	209	210	211	212
649	66	202	203	204	205	206	207	208	209	210	211	212	213	214	215
649	67	205	206	207	208	209	210	211	212	213	214	215	216	217	218
649	68	208	209	210	211	212	213	214	215	216	217	218	219	220	221
649	69	211	212	213	214	215	216	217	218	219	220	221	222	223	224
649	70	214	215	216	217	218	219	220	221	222	223	224	225	226	227
649	71	217	218	219	220	221	222	223	224	225	226	227	228	229	230
649	72	220	221	222	223	224	225	226	227	228	229	230	231	232	233
649	73	223	224	225	226	227	228	229	230	231	232	233	234	235	236
649	74	226	227	228	229	230	231	232	233	234	235	236	237	238	239
649	75	229	230	231	232	233	234	235	236	237	238	239	240	241	242
649	76	232	233	234	235	236	237	238	239	240	241	242	243	244	245
649	77	235	236	237	238	239	240	241	242	243	244	245	246	247	248
649	78	238	239	240	241	242	243	244	245	246	247	248	249	250	251
649	79	241	242	243	244	245	246	247	248	249	250	251	252	253	254
649	80	244	245	246	247	248	249	250	251	252	253	254	255	256	257
649	81	247	248	249	250	251	252	253	254	255	256	257	258	259	260
649	82	250	251	252	253	254	255	256	257	258	259	260	261	262	263
649	83	253	254	255	256	257	258	259	260	261	262	263	264	265	266
649	84	256	257	258	259	260	261	262	263	264	265	266	267	268	269
649	85	259	260	261	262	263	264	265	266	267	268	269	270	271	272
649	86	262	263	264	265	266	267	268	269	270	271	272	273	274	275
649	87	265	266	267	268	269	270	271	272	273	274	275	276	277	278
649	88	268	269	270	271	272	273	274	275	276	277	278	279	280	281
649	89	271	272	273	274	275	276	277	278	279	280	281	282	283	284
649	90	274	275	276	277	278	279	280	281	282	283	284	285	286	287
649	91	277	278	279	280	281	282	283	284	285	286	287	288	289	290
649	92	280	281	282	283	284	285	286	287	288	289	290	291	292	293
649	93	283	284	285	286	287	288	289	290	291	292	293	294	295	296
649	94	286	287												

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In August 1939 the British established the Government Code and Cipher School at Bletchley Park in Buckinghamshire. The people recruited to work there came from a variety of backgrounds. There were experienced codebreakers, secret service officers, mathematicians, scientists, crossword experts, international chess players, students, actresses and even astrologers and debutants.

Fortunately for the British codebreakers, in the years running up to the war Poland had worked on various techniques for cracking Enigma. Shortly before the German invasion of Poland, they shared their work with their British allies. Poland's government was the first to employ mathematicians as code-breakers, and the mathematicians' logical minds proved to be just what was needed to tackle Enigma.

This vital headstart from the Polish, coupled with the unique problem-solving and intuitive thinking skills of Bletchley's recruits, meant that Enigma was cracked in early 1940 a reliable technique for cracking Enigma was established. The British code breakers worked in shifts around the clock for the whole of the war, using paper and pencil as well as newly invented mechanical techniques to work out the particular Enigma machine settings for each and every single day.

Unwittingly, the Germans themselves helped the British to decipher the Enigma. For example:

- Messages often began with the same opening text – many began with the word *Spruchnummer* (Message Number), and many Air Force messages began with the phrase *An die Gruppe* (To the Group).
- Messages often enciphered routine information such as weather reports and phrases such as *Keinebesondere Ereignisse* (Nothing to report).
- Messages often ended with *Heil Hitler!*
- The Germans often transmitted the same message more than once, with each version enciphered differently.

These lapses provided the codebreakers with clues, called *cribs*, about how the Enigma machines had been set up on that day. These cribs were essential for breaking the ciphers. For example, without a crib it would still take several months today to decipher an A4 page of ciphertext using a modern PC with trial and error methods.

However, the cribs alone were not enough. The codebreakers at Bletchley Park developed new procedures and algorithms for determining the set-up of the Enigma and also had to develop electronic computing devices to implement these methods.

Today, historians believe that the work of the code breakers at Bletchley Park shortened the war by two years.

Neglected heroes

Among the most famous of the leading code breakers at Bletchley Park was a mathematician from the University of Cambridge, Alan Turing. Turing was regarded by many as a genius. He played a leading role in breaking the more complicated Naval Enigma cipher (codenamed Shark) and also established the principles behind the modern computer.

Despite their remarkable work, however, for a long time afterwards none of the second world war code breakers received the public recognition they deserved. In order to preserve British security, the breaking of Enigma remained a tightly guarded secret for the duration of the war, and for the following 30 years. The people who had worked at Bletchley Park were forbidden from talking about what they had done and as a result their contribution to the war effort was entirely forgotten. However, over the past 30 years more and

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more information has been released about the incredible story of Bletchley Park.

Tragically, for some, however, the acknowledgments have come too late. Alan Turing committed suicide before he was ever publicly recognised for his extraordinary part in the war and before his contributions to the science of codes and code breaking were fully understood.

The British Government still operates a code breaking department, at "Government Communication Headquarters" in Cheltenham. And to this day they rely on mathematicians for their problem solving abilities and logical thinking: GCHQ boasts the highest concentration of pure mathematicians in the country. Today's secret codes are much more sophisticated than the Enigma cipher and their strength relies on the inability to factorise large numbers, so with today's worries about global terrorism, the role of our code breakers is just as vital as during the second world war.

More information

- [The Enigma Project](#):
The MMP's outreach project which takes codes, code breaking and a genuine WW2 Enigma machine into the classroom.
- [Bletchley Park](#):
Find out about WW2's code breaking heroes and information on breaking Enigma. Bletchley Park is now a museum and information about visiting can also be found on their website.
- [NRICH Maths](#):
The March 2004 issue (select this on the left hand menu) is all about different secret codes with puzzles to break.
- [National Cryptologic Museum](#):
The US National Security Agency's museum of cryptography.
- [Simon Singh's Crypto Corner](#):
Information about a host of different codes. The story of the Code Book Cipher Challenge, who won it and how. Also links to other cryptography websites. Follow links to the Black Chamber for on-line puzzles and decryption tools. You can also download free copies of the Code Book CD-ROM.
- [Alan Turing Memorial](#):
Information about Alan Turing's memorial statue in Manchester.
- [Codes and Ciphers](#):
All you ever wanted to know about Second World War codes and ciphers.

Answers to the questions in the text

Answer 1

For the first slot, you can choose any one of 5 rotors. For the second, you can choose any one of 4 rotors. For the last, you can choose any one of 3 rotors. So there are

$$5 \times 4 \times 3 = 60$$

ways of positioning 5 rotors in 3 slots.

[Back to question 1.](#)

Answer 2

As there are 26 letters of the alphabet, each of the 3 rotors could be set in any one of 26 different starting positions. This gives a total of

$$26 \times 26 \times 26 = 17,576$$

distinct starting positions.

[Back to question 2.](#)

Answer 3

The first ring can be set in any of 26 positions, as can the second, so there are

$$26 \times 26 = 676$$

ways of positioning the 2 rings on a 3-rotor army Enigma.

[Back to question 3.](#)

About the author

Claire Ellis has recently joined the Millennium Mathematics Project, the Cambridge-based mathematics enrichment and dissemination group that publishes *Plus* magazine. She runs the [Enigma Project](#).



Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the [NRICH](#) and [MOTIVATE](#) sites.