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Regulars

Puzzle page

BOOKMARK

A knight's nightmare

Imagine a chess board with $n \times n$ squares, n on each side. Now imagine a knight moving around the board − only using the moves that are allowed to a knight of course − so that each square of the board is visited exactly once, and so that the knight ends up on the same square as it started. Such a tour is called a *closed knight's tour* (it's *closed* because the knight ends where it started). If you start experimenting on an ordinary

chess board, you'll soon see that it's no easy feat to find a closed knight's tour. People have been entertaining themselves with this pursuit for centuries. The earliest recorded example of a knight's tour on the ordinary 8×8 board came from al−Adli ar−Rumi, who lived in Baghdad around 840AD. There are also example of knight's tours of 10×10 and 12×12 boards.

But no−one has ever found a closed knight's tour on an $n \times n$ board when n is odd. Can you prove why this is, in fact, impossible?

If you're poetically minded, try this one: find a knight's tour on this 8×8 board, so that the syllables on the squares, when read in the sequence of the tour, form a verse (note that this time you're *not* asked for a *closed* knight's tour − it does not have to end at the same place it started).

With	white	−gle	from	−lant	black	a	star−
square	the	knight	and	sin−	−ted	gal−	of
did	nerve	And	−where	And	twice	He	−sing
prove	Nor	king's	on	it	−ny	land	A
of	once	he	back	−ting	−main	mis−	might
came	to	res−	do−	a−	to	fire	the
a−	steel	his	−gain	To	heart	−full	−out
all	a−	−spire	and	power−	With−	roam	of

The solution

Assume that the knight starts out on a white square (the argument will be the same if it starts out on a black square). Because of the way a knight moves in Chess, the next square it lands on will be black. To complete a closed knight's tour, the knight has to make $n \times n$ moves. Since n is an odd number, $n \times n$ is also odd, so the knight has to make an odd number of moves. But this means that it will end on a black square, since the colour of the square changes with each move. This is a contradiction, because the knight has to start and end on a white square.

The solution to the "cryptotour" is the verse

With nerve of steel and heart of fire
A gallant knight did once aspire
To roam the land of black and white
And prove to all his powerful might.
He started from the King's domain
And back again to it he came
Without missing a single square
Nor resting twice on anywhere.

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