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June 2006

Regulars



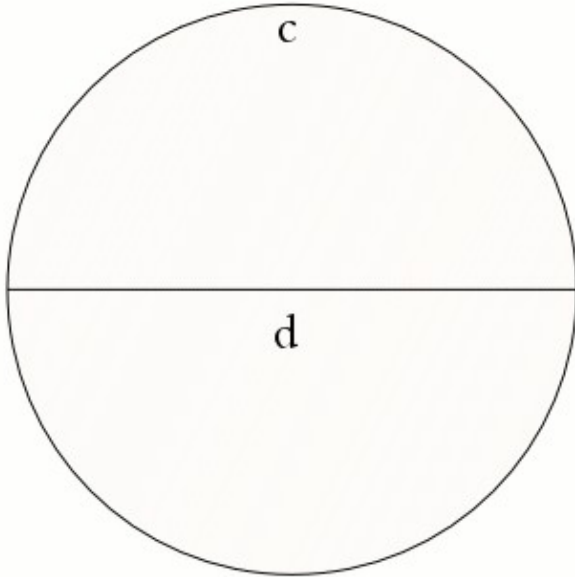
## Puzzle page



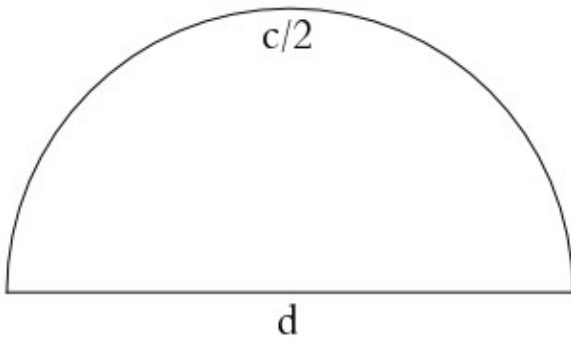
### A proof with a hole: $\pi$ equals 2

Forget all that business about  $\pi$  being an irrational number with infinitely many decimal places: I can prove conclusively that  $\pi$  is equal to 2.

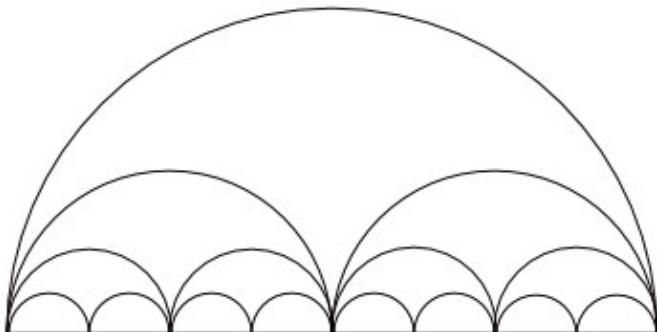
First of all, let's recall that  $\pi$  is defined to be the ratio between the circumference and the diameter of a circle, which is the same regardless of the size of the circle. So, using the notation of the diagram below, we have  $\pi = c/d$ .



Now let's start with a circle of circumference 2, and only consider one half of it, as shown in the figure. Since it's exactly one half of the circle, the length of this semi-circle is 1. Now let's divide in half the diameter  $d$  of the circle, and draw a new, smaller semi-circle on each of the two halves. Since the ratio between diameter and circumference is the same for *any* circle, you can work out that the two smaller semi-circles which are built on half the diameter of the larger one have circumference half that of the larger one. In other words, the length of each of the two smaller semi-circles is  $1/2$ .



Now continue in the same manner: divide the original diameter  $d$  into 4 equal pieces and draw on each of them a semi-circle of length  $1/4$ ; then divide it into 8 equal pieces and draw on each of them a semi-circle of length  $1/8$ , etc, etc. After  $n$  steps you have  $2^n$  semi-circles, each of length  $1/2^n$ .



## Puzzle page

Obviously, the semi-circles get smaller and smaller at each stage, and after a great number of steps, your string of semi-circles will hardly be distinguishable from the straight line which forms the diameter of the largest circle. The string of semi-circles *approximates* the diameter  $d$ , and the approximation gets better and better the more steps you take. This means that the lengths of the semi-circles all added up approximate  $d$ . In fact,  $d$  is the *limit* of this sum as the number of steps  $n$  tends to infinity:

$$d = \lim_n 2^n \times 1/2^n = 1.$$

We know that the circumference  $c$  of the large circle is 2, so  $\pi = c/d = 2/1 = 2$ , which proves my claim. Or have I made a mistake?

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If you are stumped by [last issue's puzzle](#), here is [the solution](#).

For some challenging mathematical puzzles, see the [NRICH](#) puzzles from [this month](#) or [last month](#).

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*Plus* is part of the family of activities in the Millennium Mathematics Project, which also includes the [NRICH](#) and [MOTIVATE](#) sites.