January 1998
Regulars

Solution to Puzzle No. 3 − birth dates

For the question see "Puzzle No. 3 − birth dates", in issue 3.

The answer to the problem is that, in order to share this special numerical relationship, the age of the mother must be a multiple of 9 when the child is born.

Why?

To prove that this is the case, we need a little theorem.

Theorem

If *N* is a positive integer and *S* is the sum of its digits, then *N* mod 9 = *S* mod 9.

"*N* mod 9" just means "the remainder *r* when *N* is divided by 9", where *r* can range from 0 to 8.

Proof

The proof, in a nutshell, looks like this:

If the digits of *N* are *x*[1], *x*[2],... , *x*[n] then

$$N = 10^{n-1} x_1 + 10^{n-2} x_2 + \ldots + 10 x_{n-1} + x_n$$

But it is always true that

$$10^m = \underbrace{99\ldots9}_{m-1 \text{ times}} + 1$$

So we can write:

$$N = \underbrace{99\ldots9}_{n-2} x_1 + \underbrace{99\ldots9}_{n-3} x_2 + \ldots + 9 x_{n-1} + (x_1 + x_2 + \ldots + x_n)$$

$$N = 9(\underbrace{11\ldots1}_{n-2}x_1 + \underbrace{11\ldots1}_{n-3}x_2 + \ldots + x_{n-1}) + S$$

But the first term of this expression is an exact multiple of 9.
Therefore N mod 9 = S mod 9.

QED.

This theorem tells us that summing the digits of a number does not change its value mod 9. Therefore, repeatedly summing the digits of a number until a single digit is reached does not change its value mod 9.

On the day a child is born its age is 0. Therefore, to share this special numerical relationship with its parent the parent's age mod 9 must also be 0. This is simply another way of saying that the parent's age must be a multiple of 9, e.g., 9, 18, 27, 36, 45, 54,...

Perhaps a more famous use of this theorem is in deducing that a number is divisible by 3 if and only if the sum of its digits is divisible by three. Why?
