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Regulars

Mathematical mysteries: twin primes



Prime numbers are a rich and ancient source of mathematical mystery. It has been known for over 2000 years that there are an infinite number of them.

This issue's mathematical mystery concerns pairs of prime numbers called *twin primes*. A pair of prime numbers are twins if they differ by 2. If you look at a list of the first 50 primes you'll see that it contains 16 twin prime pairs:

2 3 5 7 11 13 17 19 23 29
 31 37 41 43 47 53 59 61 67 71
 73 79 83 89 97 101 103 107 109 113
 127 131 137 139 149 151 157 163 167 173
 179 181 191 193 197 199 211 223 227 229

(3,5) (5,7) (11,13) (17,19) (29,31)
 (41,43) (59,61) (71,73) (101,103) (107,109)
 (137,139) (149,151) (179,181) (191,193) (197,199)
 (227,229)

The question is: are there infinitely many? A more general version of this question was put forward in 1850 by Alphonse de Polignac. He postulated that there were infinitely many *consecutive* primes separated by 2 (twin primes), 4, 6, etc. and by every even number.

The twin primes are used to define one of mathematics' more unusual constants. In 1919 Viggo Brun proved that the sum of the reciprocals of all the twin primes converged. The resulting constant, B , is known as Brun's constant in honour of this achievement. Despite this proof it is still not known how many twin primes there are!

$$B = \left(\frac{1}{3} + \frac{1}{5}\right) + \left(\frac{1}{5} + \frac{1}{7}\right) + \left(\frac{1}{11} + \frac{1}{13}\right) + \dots$$

As computers become more powerful so Brun's constant is known more and more accurately. In fact, in 1994 Thomas Nicely was busy revising the latest estimate when he noticed a strange discrepancy in his results. It turned out that he had discovered the now infamous Pentium processor bug. The bug was discovered only

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after more than a million PCs had been distributed with the faulty processor forcing Intel, manufacturer of the chip, to embark on a 475 million dollar replacement programme.

For more information see: [Enumeration to \$1e14\$ of the twin primes and Brun's constant](#) by Dr. Thomas R. Nicely.



Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the [NRICH](#) and [MOTIVATE](#) sites.