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Regulars



Superficiality: solution

Is it better for you to stay in one group or to split up into two or more smaller groups? This was a problem faced by naval convoys trying to avoid being found by enemy submarines during World War II, and it was also the problem posed in [last issue's Outer Space](#).

It is better to stay in a big convoy rather than to divide. Suppose that a big convoy covers an area A and the ships are as close together as they can be, so that if we divide the convoy into two smaller ones of area $A/2$ the spacings between ships are the same. The single convoy has a perimeter equal to $p = \pi\sqrt{A/\pi}$, but the total perimeter of the two smaller convoys equals $p \times \sqrt{2}$, which is bigger. So, the total perimeter distance that has to be patrolled by destroyers to protect the two smaller convoys from being penetrated by submarines is greater than that to be patrolled if it stays as a single convoy. Also, when the submarine searches for convoys to attack, its chance of seeing them is proportional to their diameter, because this is what you see in the periscope. The diameter of the single circular convoy of area A is just $\sqrt{A/\pi}$, whereas the sum of the two diameters of the convoys of area $A/2$ that don't overlap in the field of view is bigger by a factor of $\sqrt{2}$, and so the divided convoy is more likely to be detected by the attacking submarine than is the single one.

So, a single convoy is harder to detect and easier to defend than a divided one.

Further reading

The book [The Pleasures of counting](#) by TW Korner, which is [reviewed](#) in Issue 13 of *Plus*, explores more naval and aerial maths problems. [Back to Outer space](#)



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