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Regulars

Some benefits of irrationality: solution

You were asked to investigate the dimensions of the *B* paper size series. The area of a sheet of paper of size *BN* is the *geometric mean* of the areas of a sheet of size *AN* and a sheet of size *AN−1*. The geometric mean of a set of *k* numbers is defined to be the *kth* root of their product. In this case there are only two numbers involved and we get:

$$Area(BN) = L(BN)W(BN) = \sqrt{Area(AN)Area(AN-1)},$$

where *L(BN)* and *W(BN)* are the length and width of the sheet. A sheet of size *AN−1* only exists if *N−1* is greater than or equal to zero, so the formula above works for *N* greater than or equal to 1.

As we worked out before, the area of an *AN* sheet is 2^{-N} square metres, so

$$L(BN)W(BN) = (2^{-N}2^{-(N-1)})^{1/2} = (2^{-2N+1})^{1/2} = 2^{-N+1/2}.$$

The aspect ratio of the *BN* sheet is still the square root of 2, so we also get

$$L(BN)/W(BN) = 2^{1/2},$$

so

$$L(BN) = 2^{1/2}W(BN).$$

Substituting this into the equation for the area we get:

$$L(BN)W(BN) = 2^{1/2}(W(BN))^2 = 2^{-N+1/2},$$

so

$$W(BN) = 2^{-N/2} \quad and \quad L(BN) = 2^{1/2}2^{-N/2} = 2^{-N/2+1/2}.$$

Although we developed this formula to only cover the cases where *N* is greater than or equal to 1, it turns out that a sheet of size *B0* also follows its rule: it has area

$$L(B0) \times W(B0) = \sqrt{2} \times 1 = \sqrt{2}.$$

The C series of paper sizes is made from the geometric means of the same numbers in the A and B series, so C4 is the geometric mean between A4 and B4 etc.

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