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Regulars



## Some benefits of irrationality: solution

You were asked to investigate the dimensions of the  $B$  paper size series. The area of a sheet of paper of size  $BN$  is the *geometric mean* of the areas of a sheet of size  $AN$  and a sheet of size  $AN-1$ . The geometric mean of a set of  $k$  numbers is defined to be the  $k$ th root of their product. In this case there are only two numbers involved and we get:

$$\text{Area}(BN) = L(BN)W(BN) = \sqrt{\text{Area}(AN)\text{Area}(AN-1)},$$

where  $L(BN)$  and  $W(BN)$  are the length and width of the sheet. A sheet of size  $AN-1$  only exists if  $N-1$  is greater than or equal to zero, so the formula above works for  $N$  greater than or equal to 1.

As we worked out before, the area of an  $AN$  sheet is  $2^{-N}$  square metres, so

$$L(BN)W(BN) = (2^{-N}2^{-(N-1)})^{1/2} = (2^{-2N+1})^{1/2} = 2^{-N+1/2}.$$

The aspect ratio of the  $BN$  sheet is still the square root of 2, so we also get

$$L(BN)/W(BN) = 2^{1/2},$$

so

$$L(BN) = 2^{1/2}W(BN).$$

Substituting this into the equation for the area we get:

$$L(BN)W(BN) = 2^{1/2}(W(BN))^2 = 2^{-N+1/2},$$

so

$$W(BN) = 2^{-N/2} \quad \text{and} \quad L(BN) = 2^{1/2}2^{-N/2} = 2^{-N/2+1/2}.$$

Although we developed this formula to only cover the cases where  $N$  is greater than or equal to 1, it turns out that a sheet of size  $B0$  also follows its rule: it has area

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$$L(B0) \times W(B0) = \sqrt{2} \times 1 = \sqrt{2}.$$

The C series of paper sizes is made from the geometric means of the same numbers in the A and B series, so C4 is the geometric mean between A4 and B4 etc.

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