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December 2007

Features



## An almighty coincidence

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Imagine picking the four hymn numbers out of a hat. First note that four-hymn combinations with one 1-digit number and three 3-digit numbers come in four types: the one digit number can occur in first, second, third or last place of the selection. So the overall chance of picking such a combination is equal to:

- Chance of picking a combination with 1-digit number in 1st place
- + chance of picking a combination with 1-digit number in 2nd place
- + chance of picking a combination with 1-digit number in 3rd place
- + chance of picking a combination with 1-digit number in 4th place.

Each of the terms in this sum is equal to

$$9/999 \times 900/999 \times 900/999 \times 900/999,$$

so the overall chance of picking four hymn numbers such that one of them has 1 digit and the others have three digits is

$$9/999 \times (900/999)^3 \times 4 = 0.0263.$$

Now for the chance of picking a combination with two 2-digit numbers and two 3-digit numbers. There are 6 ways in which to choose the positions of the two 2-digit numbers within the string of four numbers, so this time the selection comes in 6 different types:

- 2233;
- 3322;

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- 2332;
- 3223;
- 2323;
- 3232.

The chance of picking a combination of each individual type is

$$(90/999)^2 \times (900/999)^2,$$

so the overall chance is

$$(90/999)^2 \times (900/999)^2 \times 6 = 0.0395$$

In general, the number of ways you can choose a set of  $k$  positions within a sequence of length  $n$  is

$$\frac{n!}{k!(n-k)!},$$

where  $n! = n \times (n-1) \times \dots \times 2 \times 1$ .

In our examples, we first had  $n = 4$  with  $k = 1$ , giving

$$\frac{4!}{1!3!} = 4,$$

and then  $n = 4$  with  $k = 2$ , giving

$$\frac{4!}{2!2!} = 6.$$

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