



© 1997–2009, Millennium Mathematics Project, University of Cambridge.

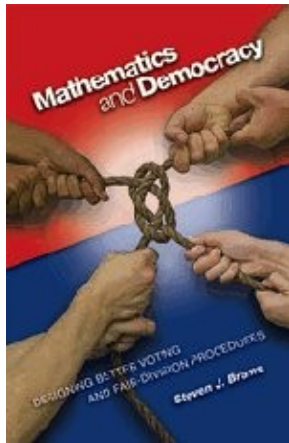
Permission is granted to print and copy this page on paper for non-commercial use. For other uses, including electronic redistribution, please contact us.

September 2008

Reviews

'Mathematics and democracy'

reviewed by Marianne Freiberger



Mathematics and democracy: Designing better voting and fair–division procedures

Steven J. Brams

We're in a US election year, and as is usual at such times there is some discussion about the fairness of the voting system. We all know what we want – democracy – the question is how to get it. But once you start thinking about the latter (and it takes a mathematician to sort out the labyrinth of possible scenarios), the former, the "what we want" part, reveals itself to be far more subtle than it first appeared. Is democracy about pleasing as many people as possible, or about offending as few as possible? Is the "voice of the people" the harmonious hum of a majority, or the squeaks and squawks of numerous minorities? Is it pure democracy we're after, or are we happy to sacrifice some of it in favour of efficiency? As this book reveals, there is no one system that fits all, and before deciding on how to extract the will of the people (or union members, or company shareholders), you need to think carefully about what you're trying to achieve.

In *Mathematics and democracy* Steven Brams makes the case for a range of new or little-known alternatives to existing democratic procedures, from electing a president to dividing up wealth. His aim is to make things more democratic and his methods are, inevitably, mathematical. If this worries you, then rest assured.

'Mathematics and democracy'

"Mathematical" in this case doesn't mean pages of horrendous equations and incomprehensible graphs. It simply means stripping a process down to its essence, following it through to its conclusion with a sober head, and trying to come up with meaningful definitions for otherwise fuzzy concepts. If you've got a head for a good logical argument and some school maths to fall back on, then you should be able to enjoy every part of this book.

The first part of the book looks at voting procedures. Brams starts off by looking at elections with several candidates and just one winner, and this is of course the class that presidential elections fall in. The draw-back of first-past-the-post systems like the one used in the US and UK, where every voter has one vote and the candidate with most votes wins, are well-rehearsed. The vote can split between two centrist candidates in favour of a more extreme one, much hangs on the decision of a few swing voters, and the system invites strategic voting and negative campaigning. Brams suggests an alternative called *approval voting*, where each voter is asked to hand in a list of candidates he or she approves of, and the candidate with most approvals wins.

Interestingly, the voters don't rank the approved candidates in order of preference they simply draw the line between who is acceptable and who is unacceptable to them. Brams makes the point that acceptability of candidates to voters is a piece of information that is missing from most other voting systems, yet in terms of making a social choice that a majority of people can live with, it may well be crucial. There are other possible advantages of approval voting that meet the eye even without deeper analysis: because voters can approve of several candidates, votes for minority candidates are not wasted and voters are not forced to vote strategically. There is little point in politicians trying to *stop* voters from voting for a rival, rather they have to *encourage* voters to include them on their approval list, so there's not much point in negative campaigning.

But common sense aside, how do you go about a mathematical analysis of such a system? The first thing you need to do is divide the voters into groups according to their preferences: group 1 consists of those voters that prefer candidate A to candidate B and candidate B to candidate C, group 2 prefer B to A and A to C, etc. With most voting systems, there is just one possible outcome given a particular preference profile: in a first-past-the-post system, for example, the winner is the candidate who is at the top of a majority of voters' preference lists. Approval voting, however, is different: the outcome depends crucially on the voters' decisions on who is acceptable to them. If all voters choose to approve only of their first choice, then the result is the same as in our first-past-the-post system. Brams shows that approval voting can in fact subsume the outcome of several other voting procedures. It is remarkably flexible and, as Brams argues, more sensitive to voters' wishes than other systems.

After examining various other features of approval voting its vulnerability to strategic voting, the stability of election results, and ways of refining it Brams moves on to elections with more than one winner, as they occur when you need to form a committee, council or legislature. Usually in this context you're after some form of proportional representation. But what exactly you mean by "proportional" hinges on what precisely it is you're trying to achieve. In some situations, for example in war-torn countries like Afghanistan where you're after a government of national unity, your main aim might be to include as many factions as possible and to antagonise as few as possible. In other situations, for example when you're looking to elect a university committee, you're less concerned with appeasing the fringes and want to focus on the moderate majority. Sometimes, for example when you're electing the board of a company, the prospect of an overly competitive election might threaten to break a previously harmonious system into antagonising factions, something you clearly want to avoid.

Brams proposes several voting procedures, some based on approval voting, to cope with this bewildering array of situations. Interestingly, his procedures can come up with diametrically opposed results, each optimising a different aspect, highlighting the fact that you really do need to think about what it is you want before deciding on a system.

'Mathematics and democracy'

Once a group of people has won power over something, attention swiftly turns to the problem of dividing the spoils between themselves. Parties in coalition governments squabble over ministries, ministers squabble over allocation of tax money, etc. Brams addresses problems like these in the second part of his book, which focuses on fair division procedures.

Again there are several aspects to be taken into account here. When forming a coalition government, stability is a major concern: there is no point in forming a coalition if members are unhappy and likely to defect. Ministries in a coalition governments need to be allocated according to their prestige and the power they carry, but also (ideally!) according to politicians' area of expertise and the opinions of the electorate. Often there is a trade-off: in economics, you need to decide whether to maximise the total amount of money in a system, typically through uncontrolled competition, or to ensure a fair distribution of wealth. When it comes to the division of power, you need to decide between a less democratic, but more efficient, central government and more democratic, but potentially wasteful and chaotic, local governments.

In seven chapters, Brams proposes and dissects a range of, often very elegant, fair division procedures pertaining to different situations. As with voting systems, the absence of a one-fits-all procedure is striking. The type of good that is to be divided, whether it's an indivisible good like government ministries, or an divisible good like money, crucially determines the procedure. What's more, there may not even be a procedure that satisfies all the criteria you could reasonably expect it to satisfy: avoiding envy and maximising efficiency, for example, seem to be locked in an eternal conflict.

It's these subtleties and complexities that justify the use of mathematics in social choice theory: no common sense argument could get to the bottom of things as thoroughly as Brams does in this book. This isn't a "popular" book in the widest sense of the word. Brams's study is thorough and structured, and he sticks to an example-proposition-proof structure that those unfamiliar with mathematics might struggle with at first. If, on the other hand, you've studied game theory before, you will meet many familiar concepts. Having said that, the book is by no means purely theoretical. Brams strengthens his arguments with a wealth of real-life examples, from US elections to the 1978 peace negotiations between Israel and Egypt. The mathematical results are amply illustrated with easy-to-follow examples, so you can still get the point even if some of the maths eludes you. In one example, Brams even comes up with a geometrical interpretation of a voting system.

Mathematics and democracy proposes alternative procedures; it isn't a survey of existing methods or a historical review of election theory. Some familiarity of standard concepts in the theory of social choice does help, but it's not essential: if you're interested in democracy, then this book makes eye-opening reading, and if you're planning on wielding power at some point in the future, then it should be compulsory!

You can find out more about approval voting and fair division procedures in Steven Brams's [Plus](#) articles [Mathematics and democracy](#) and [What do you think you're worth?](#).

Book details:

Mathematics and democracy
Steven J. Brams
paperback 390 pages (2008)
Princeton University Press
ISBN-10: 0691133212
ISBN-13: 978-0691133218

You can buy the book and help *Plus* at the same time by clicking on the link on the left to purchase from amazon.co.uk, and the link to the right to purchase from amazon.com. *Plus* will earn a small commission from your purchase.

About the author

Marianne Freiberger is Co-Editor of *Plus*.



Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.