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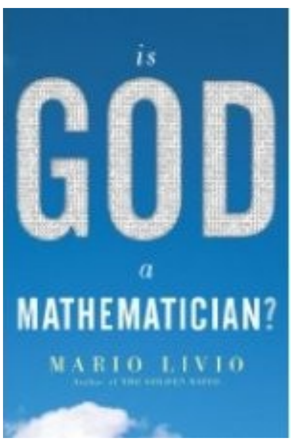
Reviews

'Is God a mathematician?'

reviewed by Marianne Freiberger



Is God a mathematician?

by Mario Livio

"Oh god, I hope not," was the reaction of a student when Livio asked the title question at a lecture, and it's a reaction that's likely to be replicated by many unsuspecting bookshop browsers. But despite its frightening title, the book's appeal could not be broader. All that's required to enjoy this fascinating read is a capacity to marvel at the mysteries of nature and humankind's attempts to understand them. This is "popular mathematics", or rather "popular philosophy", in the true sense of the word.

The question of whether god is a mathematician refers to the apparently omnipotent powers of mathematics to describe the world we live in its "unreasonable effectiveness", to use a phrase coined by physics Nobel Laureate Eugene Wigner in 1960. Evidence of this omnipotence is everywhere. The laws of physics, the movements of the stock market (though it may be hard to believe right now), the way our brain works, even chance events: all can be described in the language of mathematics. What's more, the mathematics required to solve a particular problem, for example to describe the nature of sub−atomic particles, has often been developed decades, or even centuries, before the problem was first posed. Time and time again, mathematics just happens to fit the bill perfectly. So is mathematics woven into the fabric of nature, independent of the human mind, but there for us to discover? Or is it a human invention? If it's the latter, then why does it apply to external physical phenomena at all?

These questions, as Livio points out, are not new. It was of course an ancient Greek Plato who postulated that our experienced reality is a mere shadow of an absolute world of ideas, in which ideal mathematical objects lines, circles, and so on find their rightful place. Platonist views did indeed dominate mathematical thinking for much of the long history of maths. From ancient Greece onwards (and Plato is by no means the only culprit here), many mathematicians regarded the objects of mathematics as existing independently of the human mind and spelling out an eternal truth of their own. Mathematics, then, is a matter of discovery, rather than invention. Nothing could be nobler than to pursue this truth for its own sake, and many mathematicians, including the ancient giants Pythagoras, Plato, and Archimedes, as well as modern−day mathematicians including G.H. Hardy, held a healthy disdain for those who used mathematics as a mere tool for practical problem solving, or, god forbid, profit making. Plato, as Livio tells us, thought that even astronomers should "leave the heavens alone" and concentrate on the "laws of motion in some ideal, mathematical world, for which the observable heaven is a mere illustration."

Platonism has powerful proponents even today. Livio points to the renowned mathematician Roger Penrose, who extends the mystery beyond the two−pronged vision of a world of Platonic ideas versus a world of external reality by adding a third dimension: the human mind and its conscious perception of the world. The human mind has access to the world of mathematical ideas and emerges from the physical world, which in turn seems to obey the laws of mathematics. Why is it that mathematics seems to hold this mysterious triangle together?

Mathematical Platonism certainly feels right: even dinosaurs, or aliens, could not argue with the apparent universality of the natural numbers or Euclid's ideal geometric objects. But are we being misled by a sort of observational bias? Livio quotes the famous British mathematician Michael Atiyah, who said that if we were jellyfish living in the depths of an ocean, where there are no individual objects to count, we would have built our mathematics around continuous quantities, rather than the natural numbers that appear so universal to us. Viewed in this light, mathematics really is a mere human invention, fashioned to suit our perceptions.

To throw light on his mystery, Livio takes us on a whirlwind tour of over two millennia of mathematical history, alighting at periods that were populated by the greatest mathematical innovators, including, for example, Galileo, Descartes, and Newton. The bulk of the book, framed by an introductory chapter setting out the questions and a couple of concluding chapters probing for the answers, is devoted to this history. Livio's accounts of the great lives and work are guided by the questions he's trying to answer, but they are far from dry. Historical anecdotes and interesting facts liven up the pace and we gain plenty of insight into characters, as well as theory. Archimedes, so we learn, tended to forget food and personal hygiene when working on his geometry (some things never change) and Galileo was actually unable to withstand the pressure of the church and ended up issuing an unqualified retraction of his revolutionary views on the solar system. Even if the central questions of this book leave you cold, it's worth reading simply as an introductory, if selective, mathematical history.

Throughout the book, Livio does not assume any prior knowledge of mathematics, or philosophy and history for that matter. All concepts are explained, even simple ones like prime numbers, and there's hardly an equation to be found. Even for mathematical novices, this will be a comfortable slouch−on−the−sofa read, rather than a paper−and−pencil one. A chapter on logic does require some more serious thought than the rest, but this is no surprise given the level of abstraction of the ideas Livio is trying to bring across. Those who are reasonably familiar with the history of maths may not find much that's new, but I for one still enjoyed seeing familiar episodes put into the context of the central question.

Livio's historical accounts trace the changes in the understanding of mathematics through the ages, from the mathematical mysticism of the Pythagoreans, to Newton's superhuman effort to not only explain, but also predict physical reality, and with an incredible degree of accuracy at that. As a language of the Universe that resides in a world external to the human mind, maths did indeed rule supreme for a long time, until the nineteenth century when it was hit by a major crisis.

The root of the crisis dates back to Euclid's *Elements*. This work on geometry is probably the most−read book of all time apart from the bible another illustration of the surprising longevity of mathematics. In the *Elements* Euclid set out the rules of mathematical exposition as they are still followed today: start with a set of axioms, self−evident truths that no−one in their right mind would call into doubt, and then proceed by deriving mathematical results from these axioms using only the rules of logic. Even Newton, when writing down his theory of gravitation which, after all, describes the real world rather than one of ideas, followed this strict axiomatic approach.

One of Euclid's original axioms, however, did not appear that self−evident, and for centuries mathematicians had tried to prove that it followed directly from the others, so that it could in fact be left out. In the nineteenth century a crisis ensued because it became clear not only that the axiom was independent of the others, but also that without it, you could build other geometries that were also perfectly free of contradiction. Euclid's geometry, which had been assumed to be universally woven into the fabric of nature, was only one of several self−consistent alternatives.

This devastating realisation, as Livio explains, caused a shift towards a view of mathematics as a game whose rules could be changed arbitrarily to create another game. The rules that western mathematicians throughout the centuries had chosen to adopt are by no means the only ones available, they are simply the ones that chime best with our perception of the abstract aspects of the physical world. Mathematics, then, should be viewed not only as a human product which is separate from the physical world, but also as being devoid of content: a collection of abstract objects whose interactions are governed by whatever set of rules you choose. The meat of mathematics were not Euclid's geometric shapes or Pythagoras's numbers, but the rules imposed by logic.

At the beginning of the twentieth century efforts were made to unify all of mathematics into a single abstract and axiomatic system. But a second crisis was soon to follow, as logicians proved that any axiomatic system which generates even only the simplest of mathematical objects, the natural numbers and their arithmetic, will always contain statements that no−one can ever prove to be true or false. In other words, there are things you can say of the natural numbers that are undoubtedly true, but which you'll never be able to derive from the axioms. But proving things from first principles is of course what mathematics is all about. This view of mathematics as not only a human thought experiment, but an "imperfect" one at that, is a long way from the divine truth envisioned by the likes of Plato and Descartes.

So then, is god a mathematician? After exploring what evolutionary psychologists have to say on the matter, Livio ventures some answers in his final chapter. The book is too much of a philosophical detective story for me to give the game away and tell you what they are. I'll restrict myself to an interesting point made by Livio regarding the question of whether mathematics is invented or discovered. He says it's both: mathematicians *invent* mathematical concepts, but they *discover* mathematical truths about them, because here they are guided by the rules of the game. Much like the umbrella was invented in England and not the Sahara, so was the concept of the golden ratio invented by the Greeks, and not the Indians or Chinese. The Greeks' preoccupation with geometry brought them into frequent contact with this ratio, and so they needed a name for it beyond that, there's nothing universal about this particular object. Livio's point explains a remark made by Philip David and Reuben Hersh, that mathematicians are Platonists on weekdays, and non−Platonists on Sundays. When you're doing maths, you certainly feel like you're discovering, because you're constrained by the rules of the game that's your weekday activity. But then when it comes to actually justifying your feeling that the things you've been working with all week are universal, it's easier to chicken out and pretend that you don't believe in their universality at all.

I won't give away more of Livio's own answers here. Whether or not you end up going along with them probably depends on how strong your prejudices about maths were to start with. I for one remain a convinced Platonist. But, as Livio points out, availing himself of the words of Bertrand Russell, it's not really the conclusion that counts, but the fun to be had while getting there.

To get a taste of Livio's book, read his article Unreasonable effectiveness in this issue of Plus.

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