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Regulars

## Solution to Puzzle No. 5 – complex multiplication



For the question see "[Puzzle No. 5 – complex multiplication](#)" in issue 5.

We can calculate  $p$  and  $q$  using just *three* multiplications:–

$$p = (a - b)(c + d) - ad + bc$$
$$q = ad - bc$$

the three products used being

$$ad$$
$$bc$$
$$(a - b)(c + d)$$

In fact this is not the only way to calculate  $p$  and  $q$  using three multiplications. There are many variations on the above idea, but all of them are essentially the same. If you struggled to get the solution to this issue's puzzle, why not have a go at experimenting to see how many ways you can calculate  $p$  and  $q$  using just three multiplications.

The time taken to multiply  $n$  by another number  $m$  of similar size is roughly proportional to  $(\log n)^2$ , whereas the time taken to *add*  $n$  to  $m$  is about  $\log n$ . Hence the above procedure can save a lot of computational time in any program that has to perform many complex multiplications. A good example would be the programs used to generate fractal images.

To find out more about fractals, why not see "[Modelling nature with fractals](#)", and "[The origins of fractals](#)" elsewhere in this Issue.

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Solution to Puzzle No. 5 – complex multiplication  
and MOTIVATE sites.