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Regulars

## Solution to Puzzle No. 6



For the question see "[Puzzle No 6 – world cup medallions](#)" in issue 6.

Firstly, we'll suppose that there are  $N$  different medallions to collect. This makes the working easier, plus the result we shall obtain is more general; we can simply put  $N=22$  at the end to obtain the answer to the problem.

As for [the hint](#), we define the *random variable*  $X_n = T_n - T_{n-1}$  where  $T_n$  is the number of medallions you have collected when you *first* own  $n$  different medallions.

The next step is to try to find the *distribution* of  $X_n$  i.e. to calculate [an error occurred while processing this directive]

for different values of  $n$  and  $j$  where [an error occurred while processing this directive]

Let us now fix such an  $n$  and  $j$ . In order that  $X_n = j$  we must have first picked  $j-1$  medallions which we already had in our collection of  $n-1$  different medallions, and then picked a new medallion. Assuming that each different type of medallion is equally likely to be picked at any stage (which we're told), then denoting  $p$  to be the probability that we pick a new medallion, and  $q$  to be the probability that we pick a medallion already in our collection, we have [an error occurred while processing this directive]

from which we see that [an error occurred while processing this directive]

Hence  $X_n$  has the *geometric* distribution with parameter  $p$  (defined as above). We may write this in symbols as [an error occurred while processing this directive]

It's not too hard to show that if a random variable [an error occurred while processing this directive]

then [an error occurred while processing this directive]

Hence [an error occurred while processing this directive]

The *average* number of medallions collected in total is therefore [an error occurred while processing this directive]

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and, after a little rearranging, we obtain the result: [an error occurred while processing this directive]

So, putting  $N=22$  into this formula we see that on average the number of medallions we need to collect to obtain the full set is about 81.

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