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Regulars

Solution to Puzzle No. 6

BOOKMARK

For the question see " Puzzle No 6 − world cup medallions" in issue 6.

Firstly, we'll suppose that there are N different medallions to collect. This makes the working easier, plus the result we shall obtain is more general; we can simply put N= 22 at the end to obtain the answer to the problem.

As for the hint, we define the *random variable* $X_n = T_n - T_{n-1}$ where T_n is the number of medallions you have collected when you *first* own *n different* medallions.

The next step is to try to find the *distribution* of X_n i.e. to calculate [an error occurred while processing this directive]

for different values of n and j where [an error occurred while processing this directive]

Let us now fix such an n and j. In order that $X_n = j$ we must have first picked $j-1$ medallions which we already had in our collection of $n-1$ different medallions, and then picked a new medallion. Assuming that each different type of medallion is equally likely to be picked at any stage (which we're told), then denoting p to be the probability that we pick a new medallion, and q to be the probability that we pick a medallion already in our collection, we have [an error occurred while processing this directive]

from which we see that [an error occurred while processing this directive]

Hence X_n has the *geometric* distribution with parameter p (defined as above). We may write this in symbols as [an error occurred while processing this directive]

It's not too hard to show that if a random variable [an error occurred while processing this directive]

then [an error occurred while processing this directive]

Hence [an error occurred while processing this directive]

The *average* number of medallions collected in total is therefore [an error occurred while processing this directive]

and, after a little rearranging, we obtain the result: [an error occurred while processing this directive]

So, putting $N = 22$ into this formula we see that on average the number of medallions we need to collect to obtain the full set is about 81.

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