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January 1999

Regulars

A Reader's Solution



Here is Tom Holden's solution to [puzzle number 6](#).

When you get your first coin, you are guaranteed to get a coin you have not yet got.

When you get your second coin, there is $1/22$ chance of it being one you already have, so there is a $21/22$ chance of getting one you do not yet have.

When you have two different coins, there is a $2/22=1/11$ chance of getting one you already have, so there is a $20/22=10/11$ chance of getting a new one.

When you have $n-1$ different coins (or when if you get a new coin it will be your n 'th coin), there is a $(n-1)/22$ chance of getting one you already have, so there is a $1-(n-1)/22$ chance of getting a new one.

Now the average total number of coins you need to get before you get the full set is the sum (for $i=1$ to 22) of the average number of coins you have to get before you get your i 'th different coin. So to work out the solution of the problem, we must work out how the average number of coins needed to get the i 'th different coin is affected by the number i .

This can be done easily since we know the probability of getting a coin we do not already have is $1-(i-1)/22$ and so the expected number of coins we need to take before we get a new coin is $1/(1-(i-1)/22)=22/(23-i)$ and so the total number of coins is the sum (for $i=1$ to 22) of $22/(23-i)=22$ times the sum (for $i=1$ to 22) of $1/i$ since changing the order of a sum makes no difference to its result ($a+b=b+a$).

Now the answer to the problem is given by:

$22*(1+1/2+1/3+...+1/22)=81$ to the nearest whole number of coins.

So on average, you would have to get 81 coins before you had a whole set of 22.



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Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.