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May 1999

Regulars

Letters



Syllabus information

Do you know where I could possibly get information on GCE O Level offered by Cambridge? Would be grateful if you could assist.

Michael Sital

University of Cambridge Local Examinations are now handled by OCR. Here are their details:

OCR
1 Regent Street,
Cambridge,
CB2 1GG

Tel: 01223 552552
Fax: 01223 552553
E-mail: helpdesk@ucles.org.uk
WWW: www.ocr.org.uk

A Level Statistics

I am an Open University student, studying for an MA (Education). My final project is going to be on why so many A Level mathematics students do not study maths or engineering further. Please could you assist me if you know of any articles or people I could contact regarding this issue.

Sarah Shales

Unfortunately, we don't have any figures for you here at PASS Maths. Your best bet is probably a literature search on recent educational research.

You might be interested to look at our [news story](#) in this issue, explaining the economic benefits of having an

A Level in maths.

Bottoming out on Bernoulli

I needed more information on Bernoulli's Principle and I was unable to find it on this web page. Sorry for the negative comment. You did have a lot on other topics however.

Ashley Benningfield

Sorry we didn't have the information you were looking for. Of course, PASS Maths is not a textbook, and we don't try to teach you all the complicated details of sometimes esoteric subjects. Our hope is that our articles will inspire you to follow up with further reading!

1=2?

Just thought I should point out that 1 does not necessarily equal 2 , because $2a$ doesn't equal ab unless $b = 2$. That's all.

Brian Hennessy

We're assuming that this comment refers to the example "proof" (fallacious, or course!) that $1=2$, which can be found in the [Issue 7](#) article [The origins of proof](#).

This fallacious proof starts out with the assumption that $a = b$. It never asserts that $2a$ equals ab (which, as you point out, is not true unless $b=2$).

It does assert that the **square** of a equals ab (which is obviously true if $a = b$) – perhaps your browser is not displaying the maths correctly?

Looking for problems

I am in desperate need of some Advance Level Mathematics Questions on the following topic Trigs. I Hope you can give me an ample amount of questions.

Twain Edward

Although it contains some puzzles and problems, PASS Maths is a magazine rather than a puzzle repository. There are plenty of places online where you can find puzzles: our sister site [NRICH](#) is a good place to start.

Muddled about medallions

The [World Cup medallion puzzle solution](#) is complete Greek to me. Can anyone tell me why this following approach is wrong?

Suppose I buy n packets and calculate the probability of collecting all 22 medallions is 0.5. I reason therefore that the average number of packets I need to buy is n .

Now the probability that medallion no. 1 is not in any of the n packets is $(21/22)^n$. Therefore the probability that medallion no. 1 is in at least one packet is $1 - (21/22)^n$.

I take this medallion out and now I have $n - 1$ packets left.

Now the probability that medallion no. 2 is not in any of the $n - 1$ packets is $(21/22)^{n-1}$. Therefore the probability that medallion no. 2 is in at least one packet is $1 - (21/22)^{n-1}$.

Now the probability that I've got all 22 medallions is all these terms multiplied together. Maybe not a very esoteric solution but easily done on a spreadsheet, feeding in different values of n until the product equals half and far easier to understand.

By this method I make $n = 86$, quite a close result, but seemingly not completely correct. Where am I going wrong?

R.G.

The mistake you've made with your approach is confusing the **median** with the **mean**. An example in a different context will probably help.

Suppose I put three red balls and one black ball in a bag, and draw out a ball. If it's red, I stop; if it's black, I return it to the bag and try again. How many attempts will I make on average?

- Probability of 1 attempt: $3/4$
- Probability of 2 attempts: $1/4 \times 3/4 = 3/16$
- ...
- Probability of n attempts: $3/4^n$

What is the **mean** number of attempts this would take me? That's the weighted average, i.e.

$$.1 + .2 + .3 + \dots + .n + \dots$$

which works out to be 1 and a third (i.e. $\frac{4}{3}$).

[You can check this on a calculator, or work out how to prove it by studying the "Geometric distribution" in a textbook.]

Your version of the average would probably be 1 – because the probability of 1 attempt is more than 50%. But that's a bit of a misleading answer. It always takes **at least** 1 attempt; sometimes it takes more, and the mean value being greater than 1 reflects the fact that it sometimes takes more.

When you say:

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Suppose I buy n packets and calculate the probability of collecting all 22 medallions is 0.5. I reason therefore that the average number of packets I need to buy is n .

you are actually giving the definition of the **median**. Here are some definitions:

- **Mode** – the most likely outcome (i.e. the number which has the greatest probability).
- **Median** – the outcome for which the cumulative probability is 50%.
- **Mean** – the weighted average of the outcomes.

For more information on means of probability distributions, you could read about the **expectation value** in a textbook.

Have you seen the [NRICH site](#), by the way? They have an [Ask NRICH](#) service where you can discuss things like this.

Short-changed?

You may have already heard this puzzle/mystery before, but it has still got me!!!

Three employees go into an umbrella shop and pay for three umbrellas (one each) which cost 10 pounds each. They hand over the 30 pounds and go back to work.

Ten minutes later their employer, too, needs an umbrella so goes to the same shop to buy an umbrella. When he goes up to the counter the shopkeeper says to him "three of your employees came in here earlier and brought three umbrellas. I forgot to tell them that if you buy two umbrellas you get the third half-price. I have charged them 30 pounds, they should have only paid 25 pounds. Could you please give them this 5 pounds back?"

He hands the 5 pounds to the employer. When the employer goes outside of the shop, he decides to himself that he will keep 2 pounds for himself and give each of his employees 1 pound each. This he does.

This means that each employee paid 9 pounds each for their umbrellas, costing them 27 pounds in total. Together with the 2 pounds that their employer has kept that makes a total of 29 pounds. What I do not understand is where has the other pound gone?

Please can you tell me the answer to this riddle? Thank you!!!

Rachel Street

This one is an old favourite that still gets people scratching their heads!

The muddle creeps in here because you end up counting the same money more than once, and not counting other money!

Consider the following transaction:

- Bobby gives Sally ten pounds.
- Sally gives Bobby back eight pounds.

Short-changed?

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In effect, Bobby has paid two pounds, since he gave Sally ten and got back eight. Sally has kept two pounds.

If you were to make the same mistake as in the original problem above, you would say "Bobby has paid two pounds, and Sally has kept two pounds, making a total of four pounds, so where did the other six pounds from the original ten pounds go?"

What if Bobby originally gave Sally twenty pounds, and she again kept only two and gave back eighteen? Would we then be missing **sixteen pounds** from the original twenty?!

The problem here is that you're counting the same money twice – the two pounds that Bobby paid to Sally is the same money as the two pounds that Sally has. Adding these two values together doesn't tell you anything meaningful. If you add up the *actual* moneys held at the end of the transaction – the two pounds that Sally keeps and the eight pounds that Bobby gets back – you get ten pounds, as expected.

Now, in your problem there were thirty pounds in cash originally received by the shopkeeper. At the end of the transaction, the total amount of cash in the "system" ($25 + 2 + 1 + 1 + 1$) still adds up to thirty, so there is no missing pound. In other words, all the different chunks of money add up to the original total of thirty pounds.

The fallacious missing pound creeps in when you start adding up the same money more than once.

In the story, it says that "each employee paid 9 pounds each for their umbrellas, costing them 27 pounds in total. Together with the 2 pounds that their employer has kept that makes a total of 29 pounds."

However, the two pounds that the employer has kept is part of the 27 pounds the employees have paid – it is not different money, so it can't be added to the employees' 27 pounds. You're counting it twice, just like with Bobby and Sally.

The fact that this meaningless value adds up to one less than thirty, which causes the "missing pound", is pure coincidence. If the employees had overpaid by fifteen pounds rather than five, and their boss had still kept two pounds, then by this argument you would now have eleven missing pounds:

- *Employees pay 40 pounds*
- *Shopkeeper gives boss 15 pounds back*
- *Boss keeps 2 pounds, gives employees 13*
- *Net result: Employees have paid 27 pounds, boss keeps 2, sum 29.*
- *Where's the other 11?*

In other words, by choosing the numbers properly, you can make arbitrary amounts of money "go missing", because the underlying line of reasoning is completely wrong. Sadly, this isn't going to help you on your tax return!

Cloth cutting

I have a simple problem which I can't work out. I have to cover a sphere of diameter 6 foot in 4 segments of cloth. How big does each segment have to be?

kelticore@aol.com

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The hands-on way:

1. Cut around a nice round orange starting at the stalk (the North Pole?) to the base (the South Pole?) and back to the stalk.
2. Turn the knife through a right angle and do it again.
3. Peel the orange carefully. Flatten out the segments of peel.
4. Scale the segments of peel up by the factor $6ft/(diameter\ of\ orange)$.

Voila! (p.s. allow extra for seams and errors.)

Applying some theory:

Let $R = 3ft$.

Let $S =$ width of your seams.

Then the width of cloth you need is $W = 2 \times ((\pi R) / 4 + S)$

And you'll need 4 pieces of length $H = \pi R$ (plus a bit for fiddling with).

Chalk in a centre line across the cloth.

Mark other lines at distances $H/18, 2H/18, 3H/18 \dots H/2$ above and below this line. Label these lines 10, 20,...90 above and below. They correspond roughly to lines of latitude at 10, 20,...90 degrees North and South.

Chalk in a (vertical) centre line along the length of the cloth.

To get the rough shape of the segments, measure out from the vertical line using the following table. Make two marks, one to the left and one to the right :

| | |
|----------------------|--|
| 0 (the centre-line) | $\cos(0) \times W/2$ |
| 10 (below and above) | $\cos(10) \times W/2$ |
| 20 (below and above) | $\cos(20) \times W/2$ |
| ... | ... |
| ... | ... |
| 90 (below and above) | $\cos(90) \times W/2 = 0$ (i.e. one mark at the centre-line) |

You can find the 'cos' function on a calculator. (e.g. The MS Windows calculator in Start/Programs/Accessories)

This will give you a series of tick marks you can join into a smooth curve. And if I've done my sums right it will look about the same as that segment of orange!

(P.S. You might like to think about why this second method still only gives you a good approximation to the shape you need.)

How to contact us: [Any comments?](#)



Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the [NRICH](#) and [MOTIVATE](#) sites.