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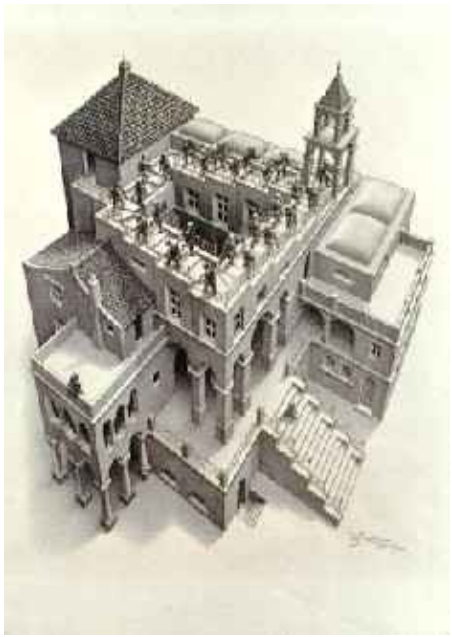
News

Count−abel even if not solve−abel

Sometimes great leaps forward in understanding come not from answering the question that is puzzling you, but from asking a different question. So it was with the ground−breaking achievement of Sir Michael Atiyah and Isadore Singer with their index theorem, which, rather than giving a way to solve a particular set of equations, gives an ingenious way to calculate the number of solutions. This may not seem immediately useful, but in fact it has provided a way to link the areas of analysis and topology, and has led to a cross−fertilization of ideas between physics and mathematics. In recognition of their achievement, Atiyah and Singer have been awarded the 2004 Abel Prize, the equivalent of the Nobel Prize in mathematics.

Mathematics is used to model many aspects of our lives, from the weather, to economics, to the electrical impulses from the heart. However, due to the complex nature of our world, finding solutions to the systems of differential equations making up a mathematical model is often very difficult.



M.C. Escher's "Ascending and Descending".

A mathematical model operates over some *space*, X, which may be related to time and space in our world. For example, a weather model might operate over a space in which each point represents a location in the atmosphere at a specific time. The *state* of the model is a series of values for each point in X that describe the phenomenon being modelled: say, the air pressure, temperature and wind speed.

These states are described mathematically as a series of functions over X, and the aim is to determine exactly what these functions are. Physical laws give information on how these states will change locally (over a small region), and these laws can be expressed as a system of differential equations. A solution to this system of differential equations would be a series of functions describing the state of the model, making it possible to explore the model over the whole space of X, predicting the weather at any point in the atmosphere at any time, say.

Finding a solution to a given system may be very difficult, or even practically impossible, but the Atiyah−Singer index theorem means that some information at least can be found. It says that the number of solutions depends essentially on the topology, or shape, of the space X over which the model operates. This means that for a system of differential equations, a researcher can check more easily if the system has zero, one or many solutions. If it has no solutions, then they know they have more work to do on the model. If the index theorem says that there is more than one solution, this information may even help them find those solutions.

John Rognes, who presented a lecture on the Atiyah−Singer index theorem at the announcement of the award, described a lighthearted illustration of the theorem. The wanderers on M. C. Escher's mysterious staircase shown in "Ascending and Descending" appear to be able to ascend (or descend) for ever. Reassuringly, you can use the index theorem to show that any model of the staircase and this motion has no solution, essentially because the topology of the square shape of the staircase does not allow a solution where the height of a wanderer is always increasing (or decreasing) − a relief to mathematicians who never like to rely just on their experience of the physical world!

The theorem, discovered in the 1960s, not only brought together topology and analysis − two very different areas of mathematics − but also started a fruitful exchange of ideas between mathematics and physics. Since they discovered the index theorem, Atiyah and Singer have tried to convey the insights of physicists to mathematicians, and also to take the tools of mathematics to the world of physics. Some of the most exciting developments in theoretical physics in recent years, such as Ed Witten's work on string theory, used mathematics derived from the index theorem.

Atiyah and Singer will receive their award, along with the £475,000 prize money, at a ceremony in Norway on 25 May. Recognising and highlighting the work of two brilliant mathematicians, and a result that, while purely mathematical, has had numerous applications and exemplifies the insights a mathematical perspective can bring, is sure to achieve the Abel's aim of showing the wider community the importance of mathematics.

More about the Abel Prize on *Plus*

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- En−Abeled

Rachel Thomas

Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.