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News

A fat chance of chaos?

BOOKMARK

I'll assume that you are familiar with complex numbers and their basic operations. If you're not have a look at *Plus* article Curious quaternions .

Let's look at the simplest quadratic polynomial $f(z) = z^2$. If you start with a complex number x_0 with modulus less than 1, for example $x_0 = 1/2$, and iterate the function, you'll slowly but surely get closer and closer to the point 0:

$$f(1/2) = 1/4, f(1/4) = 1/16, f(1/16) = 1/256, ...$$

Any other complex number within a small radius of x_0 also has modulus less than 1, and will also predictably take you to zero. A small variation of the starting value has no impact on long−term behaviour, so no butterfly effect here.

Something similar happens if you start with a complex number x_0 with modulus greater than 1, for example with $x_0 = 2$. You get the sequence

$$f(2) = 4, f(4) = 16, f(16) = 256, ...$$

The modulus of the numbers will get larger and larger; they *escape to infinity*. Exactly the same happens to all numbers within a small radius of x_0, so again no butterfly effect. But what if your starting number has modulus equal to 1? A small calculation will show you that all its iterates also have modulus 1, so they all remain on the circle with radius 1 around 0. But arbitrarily close to it there are numbers with smaller modulus which home in on zero, and numbers with larger modulus which escape to infinity. The minutest change takes you into very different directions! So the Julia set in this case is the circle of radius 1 around 0. Incidentally, this is one of the two Julia sets that is not a fractal.

Return to the article

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Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.