arctic survey education

+plus magazine

Modelling Toolkit

CLIMATE CHANGE AND THE ARCTIC ICE CAP

photograph © Martin Hartley www.martinhartley.com

A science and mathematics enrichment toolkit for ages 14-19 (Key Stages 4 and 5)

Produced by the Millennium Mathematics Project, University of Cambridge, for Arctic Survey Education

Modelling Toolkit – Contents

This enrichment toolkit explores some of the science that underlies the Catlin Arctic Survey and gives learners the chance to see curriculum science and maths applied to real-life problems. The overview article can be read on its own, or used as motivational material for the two worksheets. The worksheets are designed to promote group discussion of the topics, as well as provide hands-on activities. Because of the exploratory nature of the worksheets, less experienced students may require more guidance.

The Arctic ice cap is in trouble. Due to global warming, the Arctic sea ice cover has diminished in area as well as thickness at an alarming rate. Scientists predict that a total melt-down of the Arctic may occur within our lifetimes. But how are such predictions made? This article takes a look at mathematical models, explaining what they are and how you go about constructing one. The article can either be read on its own, as enrichment material exploring the uses of mathematics to understand natural processes, or can provide motivational material to the two worksheets listed below.

Modelling ice thickness

This activity involves constructing a simple model for the growth of sea ice, based on data from observations.

Learning outcomes: Understand the concept of a mathematical model, make predictions using a mathematical model, see how predictions vary when parameters are changed, understand the square root function.

Modelling the Earth's temperature

This activity explores a simple energy balance model of the Earth's mean temperature. The model allows students to make specific predictions of the Earth's mean temperature and of how a rise in greenhouse gases in the atmosphere and the melting of ice caps increase global warming.

Learning outcomes: Understand the concept of a mathematical model, understand energy balance models, make predictions using a mathematical model, see how predictions vary when parameters are changed, solve equations numerically and vary parameters.

Arctic Survey Education – new facts and knowledge to promote understanding

Catlin Arctic Survey

The Catlin Arctic Survey is an international collaboration between polar explorers, some of the world's foremost scientific bodies and WWF. The intention is to better predict how much time there is before the North Pole sea ice cover melts by collecting and analysing new accurate data on snow and sea ice thickness gathered by the expedition team as they journey across the Arctic.

The scientific endeavour began on 1st March 2009. The expedition was led by highly experienced polar explorer and Expedition Leader, Pen Hadow. He was accompanied by Ann Daniels, one of the world's foremost female polar explorers and Martin Hartley, the leading expedition photographer. The team travelled on foot, having hauled sledges from 81°N 130°W across drifting sea ice, for 73 days, in temperatures from 0°C to -50°C towards the North Geographic Pole.

Current estimates for the total disappearance of the Arctic Ocean's sea ice vary from 50 years down to just four. Whatever happens, the consequences of its melt-down will be of global significance in terms of sea level rise, the geo-politics of energy resources, rainfall patterns and the availability of water supplies and, of course, the impact on biodiversity, including the polar bear.

Millennium Mathematics Project – bringing mathematics to life

The Millennium Mathematics Project (MMP) is a maths education initiative for ages 5 to 19 and the general public, based at the University of Cambridge but active nationally and internationally. We aim to support maths education and promote the development of mathematical skills and understanding, particularly through enrichment activities. More broadly, we want to help everyone share in the excitement and understand the importance of mathematics. The MMP consists of a family of complementary programmes, each with a different focus, including Plus Magazine (Plus).

This enrichment toolkit was produced by Plus, a free online magazine (http://plus.maths.org) for GCSE, A level and undergraduate students, and the general public. Plus opens a door to the world of maths, with all its beauty and applications, by providing articles from the top mathematicians and science writers on topics as diverse as art, medicine, cosmology and sport. Plus provides weekly news updates, a fortnightly email newsletter, an extensive 'Careers with Maths' library and monthly podcasts.

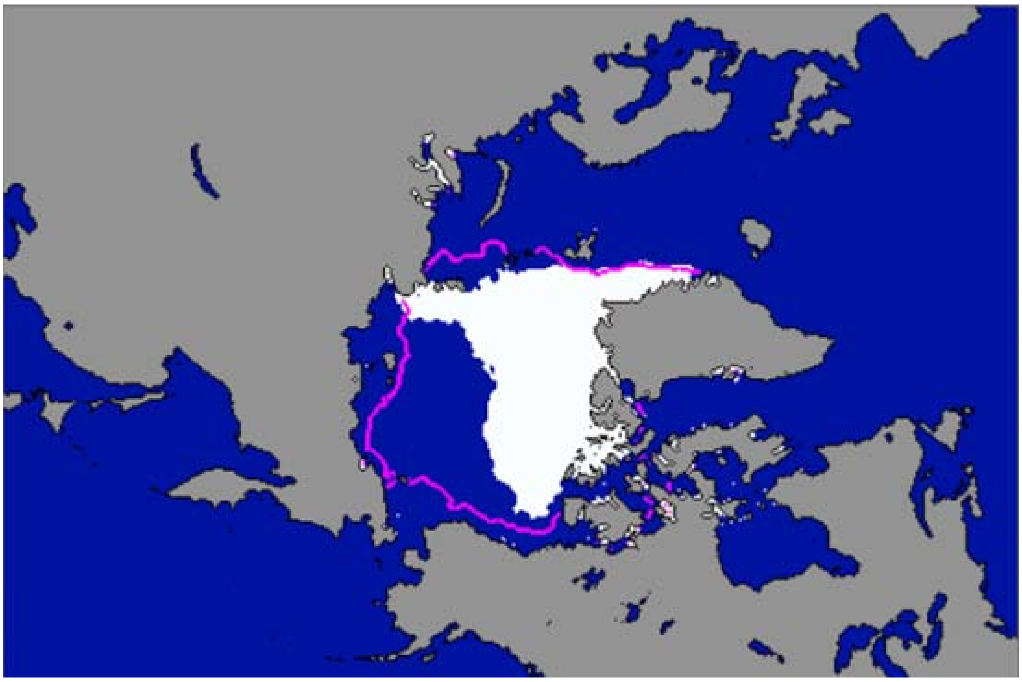
ARCTIC SUPER MODELS

The Arctic ice cap is in trouble. Due to global warming, summer sea ice cover has been shrinking by an area the size of Scotland every year. Measurements from submarines indicate that the ice has grown thinner by at least 40% over the last two decades. Predictions of if and when the permanent ice will disappear from the Arctic vary widely, but few scientists give it longer than 100 years and many predict that a total melt-down of the Arctic will occur within our lifetimes.

"All our predictions about the future of the Artic ice cap come from sophisticated mathematical models."

Peter Wadhams, Professor of Ocean Physics, University of Cambridge

Image courtesy National Snow and Ice Data Center



Sea ice extent in September 2007. The pink line indicates the average extent over the years 1979 to 2000.

But how do scientists make these predictions? A lot depends on data like that currently being gathered by the Catlin Arctic Survey. The ice team are measuring the thickness of the ice, and knowing how thick the ice is now is crucial in understanding how it will change in the future. Scientists will feed these measurements into what are known as climate models or sea ice models. These models are based on mathematics and physics, and they provide a much-needed glimpse into the future of the Arctic and the impacts of climate change.

Put simply, a mathematical model is a description of reality using equations and mathematical formulae. For example, imagine you are interested in the population of polar bears living in the Arctic. You may have observed that over the last few years, their number has halved every year. You could then use the mathematical equation:

$$y = {}^{1}/_{2}\, x$$

to describe the decline. Here x is the number of polar bears in a given year and y the number of polar bears in the following year. You can then use the equation to 'predict the future': if there are 100 bears today, then the equation tells us that there will be 50 next year, 25 the year after, and so on.

The world's favourite Arctic inhabitant

This particular model is pretty crude. For example, it doesn't take account of the fact that the population of polar bears depends on the number of ringed seals there are for them to eat. Their present decline might be due to a decline in the number of seals, and stop once the seal population has stabilised. This fact should really be included in a model – but once you start thinking about it, there is almost no end to the factors you might include. The seals themselves depend on the availability of cod, their staple diet. And what about water temperature which affects the cod population? Or human interventions like fishing?

The trick in building a mathematical model is to start simple: think carefully about the situation you're describing, and concentrate on just one or two of the most important factors. In our example, let's stick to the number of ringed seals. You then look at all the information you have, the sizes of the ringed seal and polar bear populations over the last few years, and see if you can spot a pattern. For example, you might spot that the number of polar bears seems to decline when there are less than 1000 seals and increase when the number of seals grows to more than 1000. Your model could then be improved to become:

$$y={}^{s}/_{1000}\, x,$$

where s is the number of seals. If s is greater than 1000, then $s/1000$ is greater than 1, so the polar bear population grows from year to year. If s is less than 1000, then $s/1000$ is less than 1, so the polar bear population decreases every year. This model reflects the dependence of polar bears on ringed seals.

image © Greenpeace

A ringed seal, the polar bear's favourite food.

Once you have a model you're happy with, you go back and compare it to reality. Does it match the information you have about seal and bear populations? Does it accurately predict the population size based on past bear numbers? If it does, you can assume that seal numbers really are the most important factor impacting on bear population, so you can protect the bears by protecting the seals, making sure their number remains above 1000. You can also include your model in other, more complicated, models on the whole Arctic ecosystem. If your model does not predict real data accurately, you need to introduce another layer of complexity, for example you might include the cod population and water temperature. The models used by scientists to predict animal populations are more complex than our example here, but are built up in a similar way.

As you can see, building mathematical models amounts to a subtle interplay of mathematics and observation. You observe reality, try and capture the patterns you spot using mathematics, and then go back to see if your formula really matches reality. You keep improving your model until you feel it is sufficiently accurate for your purpose.

Some mathematical models are so good that we have come to accept them as laws of nature. For example, when you give a ball a push, then the resulting acceleration of the ball is equal to the strength of the push (the force applied) divided by its mass. This rule, known as Newton's second law of motion, is so accurate that we have almost forgotten that it is really a mathematical model of reality, given by the equation:

$$F=ma$$

where F is force, a is acceleration and m is mass. We feel that this law is reality, rather than just an incredibly accurate description of reality.

Most models, and especially climate models, are far more complicated than this, consisting of many different components. In climate modelling, scientists consider equations describing the energy from the Sun and how it interacts with the Earth. For example, ocean currents like the Gulf stream can transport water that has been heated in one part of the globe to another, warming the coasts of land that would otherwise be cooler. The ice at the Poles reflects a much larger amount of the Sun's energy than open water does, so when ice cover is reduced, more energy is absorbed by the Earth and this increases the rate of global warming.

Complex mathematical models are used to predict the Earth's climate

All these factors are combined into complex large-scale models, which give the predictions you see in the newspaper headlines. Most of the time the physics and maths doesn't actually make it into these headlines, but without them no predictions would be possible.

To find out more about modelling climate change, have a go at the two worksheets accompanying this article.

Further reading:

- Maths and climate change: The melting Arctic
  http://plus.maths.org/issue46/features/wadhams/index.html
- 101 uses of the quadratic equation – a touch of quadratic chaos
  http://plus.maths.org/issue30/features/quadratic/index.html
- An introduction to energy balance models
  http://stratus.astr.ucl.ac.be/textbook/chapter3_node6.xml
- Zero dimensional energy balance models
  http://www.math.nyu.edu/caos_teaching/physical_oceanography/numerical_exercises/ebm/zero_dim_ebm.html

MODELLING ICE THICKNESS

"We believe that the Arctic ice cover will break up and disappear through thinning, rather than shrinkage. This could happen as early as 2030."

Peter Wadhams, Professor of Ocean Physics, University of Cambridge

The most visible sign of the change in Arctic sea ice is the shrinking of the area it covers, which can be seen from satellites. But scientists believe that the thinning of the ice is even more important when it comes to the future of the ice caps – they believe that the Arctic ice cover will break up and disappear through thinning, rather than shrinkage. If current trends continue this will happen as early as 2030, leaving no ice at all in summer.

photograph © Martin Hartley www.martinhartley.com

The Catlin Arctic Survey ice team moving on thin ice

But how can we predict the impact of climate change on ice thickness? The most important first step is to understand how ice grows in general. When left alone in more or less constant air and water temperatures, will a piece of ice that's floating on sea water just keep on getting thicker and thicker? Or will it eventually stabilise at a constant thickness?

To find out, a scientist working at an Arctic research station has observed a layer of ice forming on sea water for a period of two weeks. It's the Arctic winter and there is no large change in air or water temperature from day to day. Every day she measured the thickness of the ice. These are her measurements:

Time	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Thickness	1.3cm	1.8cm	2.3cm	2.6cm	3cm	3.2cm	3.4cm
Time	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 14
Thickness	3.7cm	4cm	4.1cm	4.3cm	4.5cm	4.7cm	4.8cm

Question 1: Plot the data points in a graph. What does the pattern of the points tell you about the rate of ice growth?

Question 2: Can you think of a mathematical function $f(x)$ with a shape similar to that formed by your data points?

Question 3: Plot the functions $f(x) = \sqrt{x}$ and $f(x)=1.5\sqrt{x}$ in your coordinate system. Discuss how they relate to your data points.

Based on her data, the scientist has decided to model the growth in ice thickness over time by the function

$$h(t) = c\sqrt{t}.$$

Here h denotes the thickness of the ice measured in centimetres, t is time measured in days, and c is a constant.

Question 4: Using the data, can you estimate what value c should take?

photograph © Martin Hartley www.martinhartley.com

The Catlin Arctic Survey ice team use ice drills to measure ice thickness.

Question 5: The scientist performs a similar experiment in the Arctic summer, when the temperature of air, despite still being below freezing, is a lot warmer than during the winter. How would you expect the constant c to change?

The rule we have just developed, which states that the growth of ice thickness is proportional to the square root of time, is known as the *Stefan law*, after the Austrian scientist Josef Stefan, who developed it in the 1890s.

In this example our scientist came up with the model by carefully looking at her data and finding a mathematical function that matches the data well. However it's possible to derive a very similar model directly from the laws of physics that govern the way heat is conducted in different materials. See the *Plus* magazine article *Maths and climate change: The melting Arctic* (http://plus.maths.org/issue46/features/wadhams/index.html) for more information.

This simple model only tells us what happens to ice if the air and water temperatures remain constant, and it ignores many other factors that impact on sea ice, for example the way it moves around with sea currents or is driven by the wind. It therefore does not predict the long term future of the Arctic ice cap. However, simple models like this one form the basic ingredients of the large-scale models used by scientists to predict the effects of climate change.

MODELLING ICE THICKNESS

"We believe that the Arctic ice cover will break up and disappear through thinning, rather than shrinkage. This could happen as early as 2030."

Peter Wadhams, Professor of Ocean Physics, University of Cambridge

The most visible sign of the change in Arctic sea ice is the shrinking of the area it covers, which can be seen from satellites. But scientists believe that the thinning of the ice is even more important when it comes to the future of the ice caps – they believe that the Arctic ice cover will break up and disappear through thinning, rather than shrinkage. If current trends continue this will happen as early as 2030, leaving no ice at all in summer.

photograph © Martin Hartley www.martinhartley.com

The Catlin Arctic Survey ice team moving on thin ice

But how can we predict the impact of climate change on ice thickness? The most important first step is to understand how ice grows in general. When left alone in more or less constant air and water temperatures, will a piece of ice that's floating on sea water just keep on getting thicker and thicker? Or will it eventually stabilise at a constant thickness?

To find out, a scientist working at an Arctic research station has observed a layer of ice forming on sea water for a period of two weeks. It's the Arctic winter and there is no large change in air or water temperature from day to day. Every day she measured the thickness of the ice. These are her measurements:

Time	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Thickness	1.3cm	1.8cm	2.3cm	2.6cm	3cm	3.2cm	3.4cm
Time	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 14
Thickness	3.7cm	4cm	4.1cm	4.3cm	4.5cm	4.7cm	4.8cm

Question 1: Plot the data points in a graph. What does the pattern of the points tell you about the rate of ice growth?

* ***Answer:*** *The rate of growth seems to slow down the thicker the ice becomes.*

Question 2: Can you think of a mathematical function *f(x)* with shape similar to that formed by your data points?

* ***Answer:*** *Guide students towards* $f(x) = \sqrt{x}$

Question 3: Plot the functions $f(x) = \sqrt{x}$ and $f(x)=1.5\sqrt{x}$ in your coordinate system. Discuss how they relate to your data points.

* ***Answer:*** *One lies above the data points, one lies below and they both have a similar shape.*

Based on her data, the scientist has decided to model the growth in ice thickness over time by the function

$$h(t) = c\sqrt{t}.$$

Here h denotes the thickness of the ice measured in centimetres, t is time measured in days, and c is a constant.

Question 4: Using the data, can you estimate what value *c* should take?

* ***Answer:*** *The constant c should take a value around 1.3.*

photograph © Martin Hartley www.martinhartley.com

The Catlin Arctic Survey ice team use ice drills to measure ice thickness.

Question 5: The scientist performs a similar experiment in the Arctic summer, when the temperature of air, despite still being below freezing, is a lot warmer than during the winter. How would you expect the constant *c* to change?

Answer: *It should become smaller as ice will freeze more slowly.*

The rule we have just developed, which states that the growth of ice thickness is proportional to the square root of time, is known as the *Stefan law*, after the Austrian scientist Josef Stefan, who developed it in the 1890s.

In this example our scientist came up with the model by carefully looking at her data and finding a mathematical function that matches the data well. However it's possible to derive a very similar model directly from the laws of physics that govern the way heat is conducted in different materials. See the *Plus* magazine article *Maths and climate change: The melting Arctic* (http://plus.maths.org/issue46/features/wadhams/index.html) for more information.

This simple model only tells us what happens to ice if the air and water temperatures remain constant, and it ignores many other factors that impact on sea ice, for example the way it moves around with sea currents or is driven by the wind. It therefore does not predict the long term future of the Arctic ice cap. However, simple models like this one form the basic ingredients of the large-scale models used by scientists to predict the effects of climate change.

MODELLING THE EARTH'S TEMPERATURE

The vast majority of the energy on Earth comes from the Sun, and therefore the Sun is a good starting point when it comes to building a model of the Earth's temperature and climate. A particular class of models are known as *energy balance models*. They predict the average temperature on Earth by looking at the Earth's total energy budget. So let's build our very own energy balance model and predict how greenhouse gases and the melting of the Arctic ice caps impact on temperature.

"Our predictions of the future climate of the Earth rely on sophisticated mathematical models."

Peter Wadhams, Professor of Ocean Physics, University of Cambridge

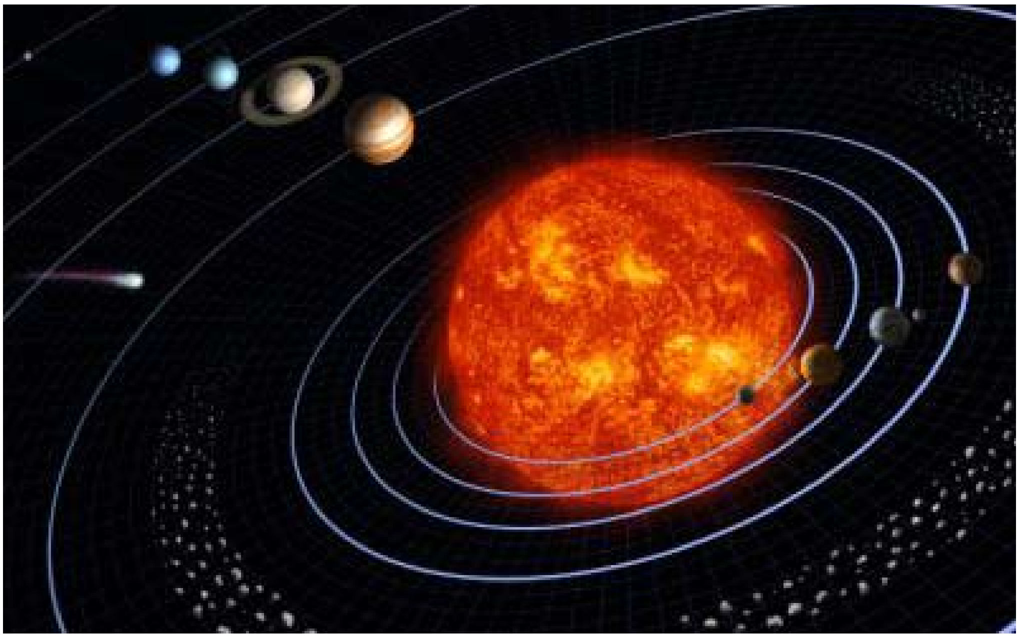


Image courtesy NASA

The Earth receives its energy from the Sun

The Sun emits energy into space in the form of radiation. About half of this energy arrives on Earth as visible sunlight, but it also includes energy from the ultraviolet and infrared parts of the spectrum, as well as microwaves and other wavelengths. Some of the solar energy is absorbed by the Earth, which in turn radiates energy back into space in the form of heat – just like an object placed near a fire heats up and starts to radiate heat into its surroundings. As the object heats up, the energy it radiates increases, until the energy radiated by the object exactly balances the energy absorbed by the object. When this happens, the object is said to be in *thermal equilibrium*. The basic assumption of energy balance models is that the Earth is also in thermal equilibrium: the amount A of solar energy it absorbs is equal to the amount R it radiates back into space.

Using measurements from satellites, scientists have been able to estimate the average amount S of solar energy that arrives on Earth per second. It is around

S=175,000,000,000 MW, which scientists write as
$$S=1.75 \times 10^{11}\ MW$$

Here *MW* stands for megawatt, and a megawatt is equal to one million watts. That's a very large number – (most coal fired power stations generate only a few hundred to a few thousand megawatts) it shows just how strong the Sun is.

The amount E of energy emitted per second by the Earth is related to the average temperature on the Earth's surface. A physical law known as the *Boltzmann law* allows scientists to estimate this relationship:

$$E=29T^4$$

where T is temperature measured on the Kelvin scale. (To convert Kelvin into degree centigrade, simply subtract 273.15.)

Now assuming that the Earth absorbs all the energy that arrives from the Sun (so $S=A$), and that all of this is emitted back into space (so $E=R$), our energy balance assumptions gives the equation

$$1.75 \times 10^{11} = 29T^4$$

Question 1: Solve this equation to find possible values for the Earth's average temperature *T*. Look up the real average temperature of the Earth – how does our model's prediction compare to this real value?

Question 2: Discuss how the model could be made more sophisticated.

photograph © Martin Hartley www.martinhartley.com

The vast areas of Arctic ice reflect sunlight and increase the Earth's albedo

In our simple model we have made two important assumptions. The first assumption was that all the energy arriving at the Earth is actually absorbed by the Earth. But this isn't quite true – some of the energy is reflected right back into space, especially when it hits white areas like the polar ice caps. The proportion a of the solar radiation that is reflected back into space is called the Earth's *albedo*. It is roughly equal to $a=0.32$. The proportion of solar radiation that is not reflected back into space is $(1-a)$. Consequently, the total average amount A of solar radiation absorbed by the Earth is

$$A=(1-a)S.$$

Any climate model must take into account the Earth's atmosphere

The second assumption we made is that all the energy radiated by the Earth makes it back into space. But this ignores the fact that the Earth has an atmosphere in which energy can get trapped. In reality, greenhouse gases like water vapour, carbon dioxide, methane and ozone exist in the atmosphere and absorb energy. This leads to the well-known greenhouse effect, which is such an important factor in our climate, and which keeps us warm. The ratio e comparing the radiation that is actually emitted by the Earth and the amount it would emit if there was no atmosphere is called the Earth's *emissivity*. It is roughly equal to $e=0.61$, showing that the amount of radiation emitted by the Earth is reduced by the presence of our atmosphere – that is, the atmosphere is absorbing energy.

Therefore, the amount R of energy radiated by the Earth into space is

$$R=e\,E.$$

Since $A=R$, we get

$$(1-a)\,x\,S = e\,x\,29T^4$$

Question 3: Solve this equation to find new possible values for *T* and compare to observed values.

Question 4: Now imagine that the temperature on Earth has risen, causing a large part of the Arctic ice caps to melt. What does this mean for the Earth's albedo, that is, the amount of sunlight that is reflected back into space, rather than being absorbed? How does this change in albedo in turn impact on temperature?

Question 5: Suppose that the Earth's albedo has decreased to $a=0.25$. How does this affect the average temperature as predicted by the model? How does this result tally with your answer to the previous question?

Question 6: Now imagine that the amount of greenhouse gases in the Earth's atmosphere increases. What does that mean for temperature? What happens to the Earth's emissivity?

Question 7: Suppose the Earth's emissivity has decreased to $e=0.55$. How does this effect temperature as predicted by our model (assuming that albedo remains the same at $a=0.32$)?

As we've seen, our simple model accounts for two very important factors: the Earth's albedo and the effect of greenhouse gases. It also makes reasonably accurate predictions about the average temperature on Earth. This simple energy balance model forms the basis of many, more complex climate models.

More sophisticated energy balance models also take into account geographical variations of temperature on Earth – the Earth is divided into areas according to geographical features, for example oceans or mountain ranges, and individual models are built for these regions. There is almost no end to the layers of complexity that can be added to models like these – for example one can take into account the different layers of the Earth's atmosphere and circulation systems like the Gulf stream. However, simple models like the one we developed here are often used as basic components of more sophisticated models.

MODELLING THE EARTH'S TEMPERATURE

The vast majority of the energy on Earth comes from the Sun, and therefore the Sun is a good starting point when it comes to building a model of the Earth's temperature and climate. A particular class of models are known as *energy balance models*. They predict the average temperature on Earth by looking at the Earth's total energy budget. So let's build our very own energy balance model and predict how greenhouse gases and the melting of the Arctic ice caps impact on temperature.

"Our predictions of the future climate of the Earth rely on sophisticated mathematical models."

Peter Wadhams, Professor of Ocean Physics, University of Cambridge

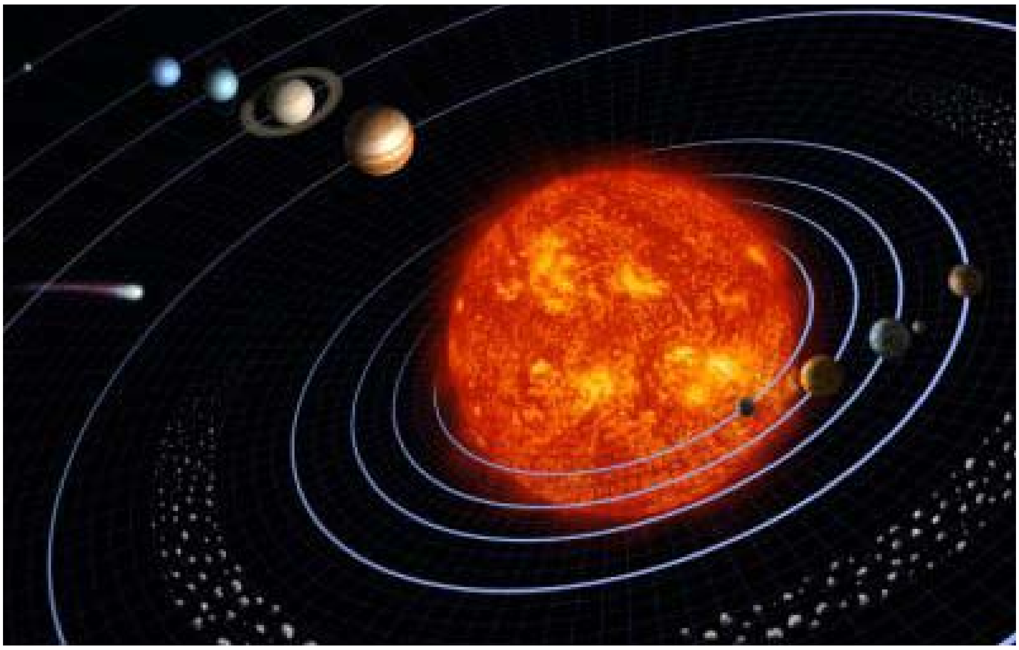


Image courtesy NASA

The Earth receives its energy from the Sun

The Sun emits energy into space in the form of radiation. About half of this energy arrives on Earth as visible sunlight, but it also includes energy from the ultraviolet and infrared parts of the spectrum, as well as microwaves and other wavelengths. Some of the solar energy is absorbed by the Earth, which in turn radiates energy back into space in the form of heat – just like an object placed near a fire heats up and starts to radiate heat into its surroundings. As the object heats up, the energy it radiates increases, until the energy radiated by the object exactly balances the energy absorbed by the object. When this happens, the object is said to be in thermal equilibrium. The basic assumption of energy balance models is that the Earth is also in *thermal equilibrium*: the amount A of solar energy it absorbs is equal to the amount R it radiates back into space.

Using measurements from satellites, scientists have been able to estimate the average amount S of solar energy that arrives on Earth per second. It is around

S=175,000,000,000 MW, which scientists write as
$$S=1.75 \times 10^{11} \text{ MW}$$

Here *MW* stands for megawatt, and a megawatt is equal to one million watts. That's a very large number – (most coal fired power stations only a few hundred to a few thousand megawatts) it shows just how strong the Sun is.

The amount E of energy emitted per second by the Earth is related to the average temperature on the Earth's surface. A physical law known as the *Boltzmann law* allows scientists to estimate this relationship:

$$E=29T^4$$

where T is temperature measured on the Kelvin scale. (To convert Kelvin into degree centigrade, simply subtract 273.15.)

Now assuming that the Earth absorbs all the energy that arrives from the Sun (so $S=A$), and that all of this is emitted back into space (so $E=R$), our energy balance assumptions gives the equation

$$1.75 \times 10^{11} = 29T^4$$

Question 1: Solve this equation to find possible values for the Earth's average temperature *T*. Look up the real average temperature of the Earth – how does our model's prediction compare to this real value?

*

Answer: *T is around 279K or -279K. The latter is physically impossible because it is below absolute zero, so we discard this solution. The former translates to around 6˚C. This is quite a lot below the observed value for average temperature, which is around 14˚C.*

photograph © Martin Hartley www.martinhartley.com

The vast areas of Arctic ice reflect sunlight and increase the Earth's albedo

Question 2: Discuss how the model could be made more sophisticated.

***Answer:** Guide students towards albedo and emissivity of the Earth. See below for an explanation.*

In our simple model we have made two important assumptions. The first assumption was that all the energy arriving at the Earth is actually absorbed by the Earth. But this isn't quite true – some of the energy is reflected right back into space, especially when it hits white areas like the polar ice caps. The proportion a of the solar radiation that is reflected back into space is called the Earth's *albedo*.

It is roughly equal to $a=0.32$. The proportion of solar radiation that is not reflected back into space is $(1-a)$. Consequently, the total average amount A of solar radiation absorbed by the Earth is

$$A=(1-a)S.$$

Any climate model must take into account the Earth's atmosphere

The second assumption we made is that all the energy radiated by the Earth makes it back into space. But this ignores the fact that the Earth has an atmosphere in which energy can get trapped. In reality, greenhouse gases like water vapour, carbon dioxide, methane and ozone exist in the atmosphere and absorb energy. This leads to the well-known greenhouse effect, which is such an important factor in our climate, and which keeps us warm. The ratio e comparing the radiation that is actually emitted by the Earth and the amount it would emit if there was no atmosphere is called the Earth's *emissivity*. It is roughly equal to $e=0.61$, showing that the amount of radiation emitted by the Earth is reduced by the presence of our atmosphere – that is, the atmosphere is absorbing energy.

Therefore, the amount R of energy radiated by the Earth into space is

$$R=e\,E.$$

Since $A=R$, we get

$$(1-a)\times S = e \times 29T^4$$

Question 3: Solve this equation to find new possible values for T and compare to observed values.

* ***Answer:*** T=286K, *which is 13˚C – much better agreement with observations.*

Question 4: Now imagine that the temperature on Earth has risen, causing a large part of the Arctic ice caps to melt. What does this mean for the Earth's albedo, that is, the amount of sunlight that is reflected back into space, rather than being absorbed? How does this change in albedo in turn impact on temperature?

* ***Answer:*** *The albedo will decrease, as less sunlight is reflected. This means that more sunlight is absorbed by the Earth, causing temperature to rise. This is known as the ice-albedo feedback loop: the more ice melts, the warmer the Earth becomes, causing yet more ice to melt, etc. For reference, the albedo of pure ice is 0.9, meaning that it reflects 90% of the sunlight.*

Question 5: Suppose that the Earth's albedo has decreased to $a=0.25$. How does this affect the average temperature as predicted by the model? How does this result tally with your answer to the previous question?

* ***Answer:*** *The average temperature increases to 293K or 20˚C. The model predicts the ice-albedo feedback loop.*

Question 6: Now imagine that the amount of greenhouse gases in the Earth's atmosphere increases. What does that mean for temperature? What happens to the Earth's emissivity?

* ***Answer:*** *The temperature will rise as more energy remains in the Earth's atmosphere. The Earth's emissivity decreases.*

Question 7: Suppose the Earth's emissivity has decreased to $e=0.55$. How does this effect temperature as predicted by our model (assuming that albedo remains the same at $a=0.32$)?

* ***Answer:*** *The temperature increases to around 294 K, which is equal to 21˚C.*

As we've seen, our simple model accounts for two very important factors: the Earth's albedo and the effect of greenhouse gases. It also makes reasonably accurate predictions about the average temperature on Earth. This simple energy balance model forms the basis of many, more complex climate models.

More sophisticated energy balance models also take into account geographical variations of temperature on Earth – the Earth is divided into areas according to geographical features, for example oceans or mountain ranges, and individual models are built for these regions. There is almost no end to the layers of complexity that can be added to models like these – for example one can take into account the different layers of the Earth's atmosphere and circulation systems like the gulf stream. However, simple models like the one we developed here are often used as basic components of more sophisticated models.

Acknowledgments

Arctic Survey Education would like to thank its Founder Members and the members of the Advisory Panel for supporting the development of this education resource.

Founder Members as at May 2009

Lloyd's of London
Aspen Re
Business in the Community and The Prince's May Day Network
Catlin Group
Flagstone Re
Linklaters LLP
Miller Insurance Services Limited
Newton Abbot College
Rix & Kay Solicitors LLP
Royal Geographical Society with IBG
United Utilities plc
University of Cambridge Department of Applied Mathematics and Theoretical Physics
Willis Group
WWF

■ = Major Donors

Advisory Panel as at May 2009

Arctic Survey Education is guided by an Advisory Panel. Members review, comment upon and strengthen Arctic Survey Education resources.

Panel members act in a voluntary and independent capacity.

Katie Webber
Campaign Manager – Environment, Business in The Community

Pete Davis
Public Engagement Team – Education, Oxfam

Katie Marchant
Deputy Head Teacher, St Angela's Ursuline Secondary School, Newham

Steve Brace
Head of Education and Outdoor Learning, The Royal Geographical Society with IBG

Professor Peter Wadhams
Polar Ocean Physics Group, Department of Applied Mathematics and Theoretical Physics, University of Cambridge

Karen Gates
Schools and Community Relations Manager, WWF-UK

Alexandra Hartridge
Climate Change Communications Manager, WWF-UK

Arctic Survey Education concept and development by James Forte www.vision40.com

Arctic Survey Education is a trading name of, and Catlin Arctic Survey is an event of, Pen Hadow Consulting Ltd.

Designed by Lydia Thornley www.thornley.co.uk