A Once-a-Century Double Consecutive *e* Days to Celebrate Number *e*

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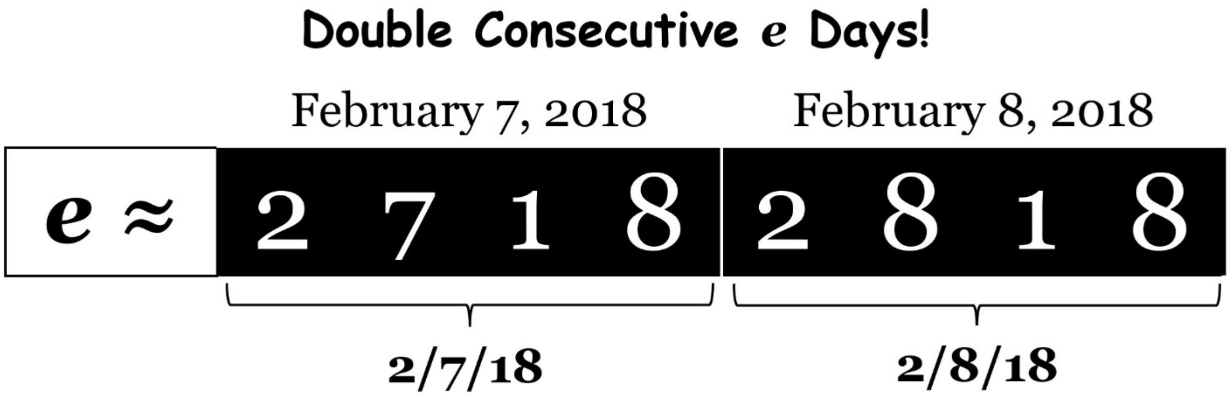

The number *e* is an irrational mathematical constant, approximately equal to 2.7182818, which pops up in many different settings throughout mathematics [1]. It was discovered by Swiss mathematician Jacob Bernoulli in 1683 while he was studying compounded interest equally divided over a period of *n* intervals which led to the total value being proportional to $(1 + 1/n)^n$ and Bernoulli noticed that this expression approaches the value of *e* in the limit *n* goes to infinity. At the time, letter *e* was not assigned to this constant.

Swiss mathematician Leonhard Euler (1707-1783) introduced the letter *e* to represent this constant in 1727 or 1728 in an unpublished paper on explosive forces in cannons and the first appearance of *e* in a publication was in Euler's *Mechanica* published in 1736. Euler calculated *e* up to 23 decimal places and his choice of the symbol *e* is said to be retained in his honor. This number is also called Euler's number.

The number *e* holds a very important place in mathematics alongside with constants 0, 1, π, and *i* where interestingly, all five of these numbers appear in an equation called Euler's Formula given by $e^{i\pi} + 1 = 0$. Additionally, the value of *e* can also be calculated as the sum of the infinite series $e = \sum_{n=0}^{\infty} 1/n!$

Just like Pi Day which occurs on March 14 every year since March 14 expressed in month/day date format as 3/14 coincides with the first three digits of π, to celebrate number *e*, February 7 expressed as 2/7 is called *e* Day because 2/7 constitutes the first two digits of *e*. However, in the day/month date format, *e* Day is celebrated every year on 2 July instead of February 7.

e Day in 2018 expressed in the month/day calendar date format as 2/7/18 was a special *e* Day because 2718 represents the first four digits of *e* and this coincidence occurs only once a century. Furthermore, I observed that the calendar date that follows 2/7/18 in the month/day date format, namely February 8, 2018, expressed as 2/8/18, coincides with the next four digits of *e*. So, these double-consecutive calendar dates in 2018, February 7 and 8, expressed as 2/7/18 and 2/8/18 put side by side as 27182818 constitutes the first eight digits of *e*. This property makes the once-a century occurring special *e* Day 2/7/18 even more special because these double-consecutive *e* Days 2/7/18 and 2/8/18 side by side represent the first eight digits of *e*.

The once-a-century occurring double consecutive *e* Days intrigued my interest regarding the early digits of number *e*. After carrying out further investigation, I would like to report the following interesting arithmetical properties about the early digits of *e* in which I ignored the decimal point between the first two digits of *e*, namely 2 and 7:

Table 1: The first 42 digits of *e*.

1st-6th	7th-12th	13th-18th	19th-24th	25th-30th	31st-36th	37th-42nd
2.71828	182845	904523	536028	747135	266249	775724

1. Half of the first six digits of number *e* given by 271828 results in 135914 and interestingly, reordering these digits as 314159 yields the first six digits of π.
2. Further, if 271828 is split as 27 and 1828, 1828 minus 27 square equals the sum of 271 and 828, the left and right halves of 271828.
3. Number 1828 repeats consecutively as the 3rd to 6th and the 7th to 10th digits of *e* (2.718281828). Interestingly, the sum of the digits of 1828 equals 19, the reverse of 19 is 91, the square of 91 equals 8281, and 8281 is the reverse of 1828.
4. Also, if 1828 is split in the middle as 18 and 28, the sum of these two numbers equals 46 and if 46 is split as 4 and 6, the 4th and 6th prime numbers are 7 and 13 and 7 times 13 equals 91. Also, 46 minus the reverse of 91 (19) equals 27, the first two digits of *e*.
5. Additionally, 91 minus 19 equals 72 and 72 is the reverse of 27, the first two digits of *e*.
6. Moreover, 19 is the 8th prime number and 19 plus 8 yields 27, the first two digits of *e*.
7. The 2nd to 4th digits of *e*, namely 718, equals 2 times 359 where these two prime numbers add up to 361 and 361 equals 19 square. Note also that 359 is the 72nd prime number where the reverse of 72, namely 27, is again the first two digits of *e*.
8. Furthermore, if 1828 is again split as 18 and 28, the sum of the reverses of these two numbers, namely 81 and 82, results in 163 and the reverse of 163 is 361, that is, 19 square.
9. Note also that 163 is the 38th prime number and 38 equals twice 19. Also, the 38th day of each year is 2/7, *e* Day in the month/day date format.
10. The sum of the prime factors of 1828, namely 2 and 457, yields 459 which constitutes the 11th to the 13th digits of *e*, following its first ten digits as 2.718281828459.
11. Note also that 459 plus its reverse, namely 954, results in 1413, the reverse of the first four digits of π.
12. Number 828459045 represents the 8th to 16th digits of *e*. If this number is split as 828, 459, and 045, there is a simple arithmetical connection between these three three-digit numbers: twice the difference of the reverses of 459 and 045, namely 954 and 540, equals 828.
13. If the 3rd to 14th digits of *e*, namely 182818284590, is split as 1828, 1828 and 4590, 4590 minus twice 1828 equals twice 467. Interestingly, 467 is the 91st prime number and again, 91 square equals 8281, the reverse of 1828.
14. The 13th to 15th digits of *e* given by 904 divided by 2 results in 452, the 15th to 17th digits of *e*. Further, if the first five digits of *e*, namely 27182, is split as 27 and 182, 182 plus the reverse of 27, namely 72, equals 254 and 254 is the reverse of 452.

15. Moreover, if the first 14 digits of *e* are split in groups of two-digit numbers as 27, 18, 28, 18, 28, 45, and 90, respectively, the sum of these numbers equals 254.
16. Also, 254 times 5 divided by 2 yields 635 and 635 is the reverse of 536, the 19th to 21st digits of *e*.
17. If the first six digits of *e* given as 271828 is split as 27, 18, and 28, the sum of the reverses of these three numbers, namely 72, 81, and 82, equals 235, the 17th to 19th digits of *e*.
18. One third of the sum of 523 and 536, the 16th to 18th and 19th to 21st digits of *e*, equals 353, the 18th to 20th digits of *e*. Note that 353 also equals the sum of 271 and 82, which side by side constitute the first five digits of *e*.
19. The difference between the reverses of 235 (17th to 19th digits of *e*) and 271 (1st to 3rd digits of *e*), namely 532 and 172, equals 360, the 20th to 22nd digits of *e*.
20. Also, the difference between 459 (11th to 13th digits of *e*) and the reverse of 271 (1st to 3rd digits of *e*) yields 287, the 23rd to 25th digits of *e*.
21. The difference of 1828 (3rd to 6th and 7th to 10th digits of *e*) and the reverse of 459 (11th to 13th digits of *e*), namely 954, equals 874, the 24th to 26th digits of *e*.
22. Half of the sum of the reverses of 459 (11th to 13th digits of *e*) and 045 (14th to 16th digits of *e*), namely 954 and 540, equals 747, the 25th to 27th digits of *e*.
23. Further, half of the reverse of 235 (17th to 19th digits of *e*), namely 532, equals 266, the 31st to 33rd digits of *e*.
24. The difference between 353 (18th to 20th digits of *e*) and 602 (21st to 23rd digits of *e*) equals 249, the 34th to 36th digits of *e*. Moreover, 249 equals one third of 747 (25th to 27th digits of *e*).
25. Additionally, 249 equals the difference of 536 (19th to 21st digits of *e*) and 287 (23rd to 25th digits of *e*). Also, twice 287 minus 249 equals 325, which is the reverse of 523 (16th to 18th digits of *e*).
26. The sum of 028 (22nd to 24th digits of *e*) and 747 (25th to 27th digits of *e*) equals 775, the 37th to 39th digits of *e*.
27. Twice 271 (1st to 3rd digits of *e*) plus 182 (3rd to 5th digits of *e*) equals 724, the 40th to 42nd digits of *e*.

I hope these properties serve to generate more interest in the digits of number *e*. Further, some of these properties may also possess the potential to someday help mathematicians and scientists to uncover the mystery of number *e*. Lastly, I wish future generations will recognize and celebrate the Double Consecutive *e* Days in every year ending with 18, with the first one to occur in 2118.

[1] *e* (mathematical constant), Wikipedia
https://en.wikipedia.org/wiki/E_(mathematical_constant)