UNIVERSITY OF CAMBRIDGE

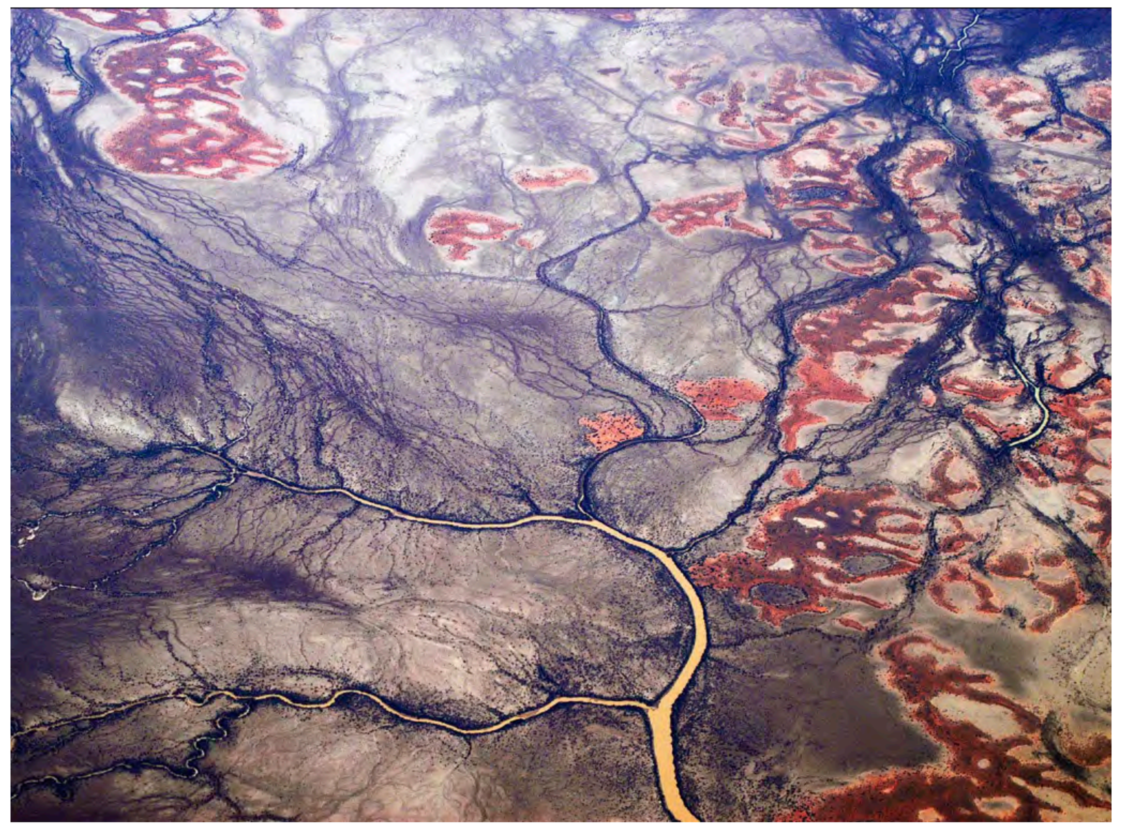
Universal pictures

Professor Peter Markowich is head of the Applied Partial Differential Equations (PDEs) research group. He's also a photographer, and his pictures below illustrate some of the applications of PDEs to physics, engineering, biology and socio-economics.

Turbulence

The image above shows the turbulent waters of the Iguassu Falls on the border of Brazil and Argentina. Turbulence in liquids and gases is one of the most complex phenomena in nature. Yet understanding turbulence is essential in anything from aircraft engineering to understanding blood flow.

In mathematics turbulence is described by a set of partial differential equations (PDEs) known as the *Navier-Stokes equations*, whose solutions give the velocity and pressure of the fluid at each point. But exact solutions exist only for very simple flows. In fact, no one knows if there even are solutions in the most general case - that's one of the biggest open questions in mathematics. For practical purposes, scientists use numerical methods to find approximate solutions.



The optimal path

The image above shows a branching river in south eastern Australia. The branching pattern is formed as the river seeks an optimal path through the landscape.

The problem of optimal transportation (in this case the transportation of water) was first explored in the late 18th century, when the French engineer Gaspar Monge was considering the unglamorous problem of how to best shift a pile of building rubble into a hole.

In the 19th century it became clear that optimal transportation problems can be described in terms of PDEs. Today optimal transportation principles are used to model the branching of rivers and trees, the veins within a leaf, and they have even been applied to urban planning, to find out how best to transport people and resources.

Spots and stripes

When it comes to animal patterns, nature regularly outdoes herself. In this area too, PDEs describe what's going on. The mathematical model which describes animal pattern formation was devised by the father of modern computer science, Alan Turing.

Turing was looking at systems consisting of two components, which can react with each other and diffuse. Imagine that the two components are chemicals in the skin of an animal embryo. One of them is an *inhibitor*, which suppresses the production of both itself and the other chemical, and the other an *activator*, which promotes the production of both chemicals. Turing's reaction-diffusion equations describe the interaction of the chemicals. They show that under certain circumstances, their concentrations will break up into evenly spaced patches.

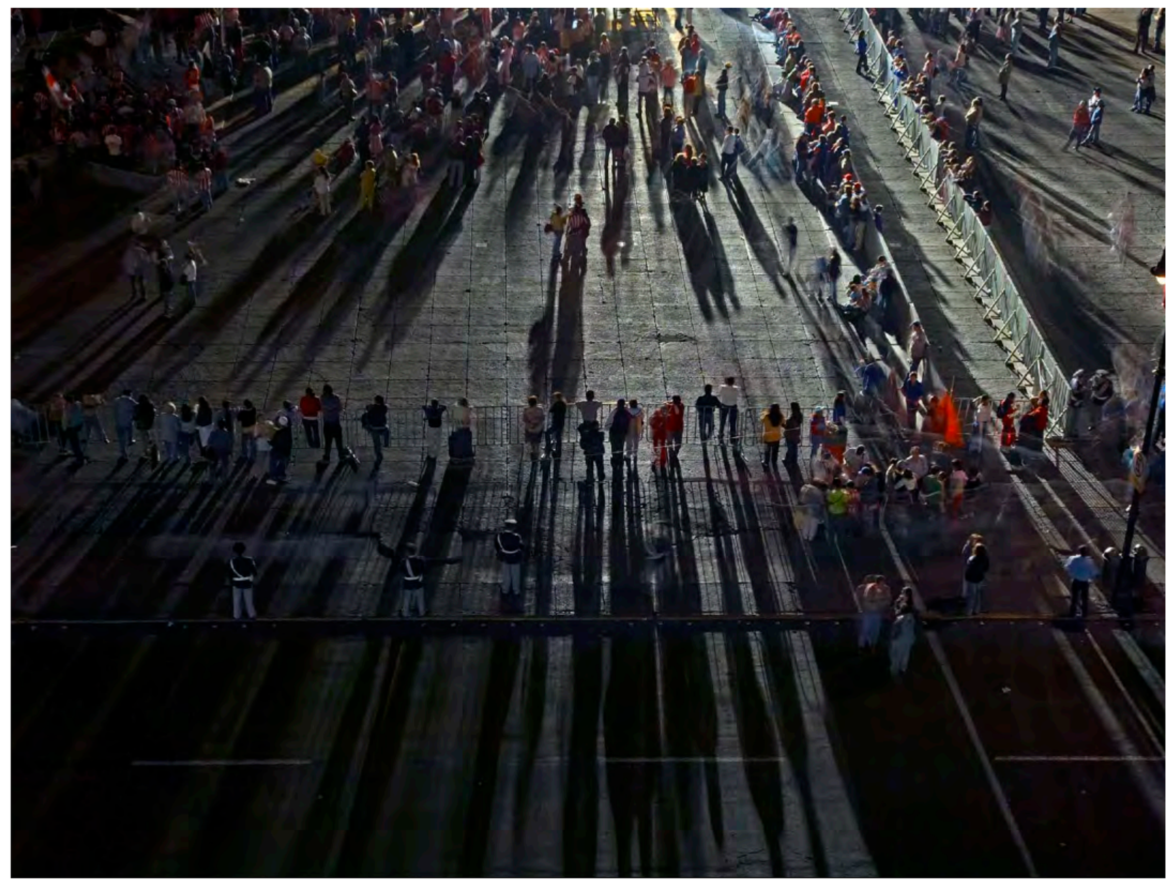
If the activator also promotes the production of a pigment, then we get the familiar patterns of animal coats.

People

Over the last ten years mathematics has started to hit the sociological sciences. One application of PDEs involves opinion formation. People meet and discuss something, and generally they come out of a discussion with slightly changed opinions. This is reminiscent of the behaviour of atoms in a gas: two atoms collide, and they come out of the collisions with slightly changed velocities.

The behaviour of atoms in a gas is described by a PDE known as the *Boltzmann equation*, and Boltzmann-like equations have been used to model opinion forming. Other important factors that impact on opinion formation, like the presence of media or of strong leaders, can be built into these models. The results have been used to understand the behaviour of gangs in LA.

The image below shows opinion forming in action in Mexico City.



This poster is adapted from the article "Universal Pictures", based on an interview with Professor Peter Markowich from the Department of Applied Mathematics and Theoretical Physics, University of Cambridge. All images used by kind permission of Professor Peter Markowich. The article is published in Plus (plus.maths.org), a free online magazine opening a door to the fascinating world of mathematics. Plus is part of the University of Cambridge's Millennium Mathematics Project.