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Features



## Euler's polyhedron formula

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### No simple polyhedron has seven edges.

**Proof:** We show first that for any polyhedron we have  $2E \geq 3F$  and  $2E \geq 3V$ . The faces of the polyhedron are polygons, each bounded by a number of sides. Along each edge exactly two faces come together, so an edge corresponds to exactly two sides: the total number of sides is  $2E$ . We also notice that any face has *at least 3* sides, so the total number of sides is *at least 3* times the number of faces. Thus we get:

The total number of sides =  $2E$

and

The total number of sides  $\geq 3F$ .

Putting this together we get:

$$2E \geq 3F,$$

proving our first inequality.

To prove the second inequality we count the total number of *ends* of edges. Each edge has two ends, so the total number of ends equals  $2E$ . At each vertex at least three edges come together, so the total number of ends of edges is at least 3 times the number of vertices. Putting this together we get:

## Euler's polyhedron formula

$2E = 3V$ .

Now, if a polyhedron has 7 edges, then  $3F = 14$  and  $3V = 14$ . This means that both  $F$  and  $V$  cannot be bigger than 4. A little thought will convince you that every polyhedron has strictly more than three faces, so we must have  $F=4$ . Similarly we get that  $V=4$ . This gives

$$V - E + F = 4 - 7 + 4 = 1 \neq 2.$$

This tells us that our hypothetical seven-edged polyhedron cannot exist, for if it did, then Euler's formula would hold and the above sum would have to be equal to 2 QED!

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