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Features

Euler's polyhedron formula

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No simple polyhedron has seven edges.

Proof: We show first that for any polyhedron we have $2E\ \ 3F$ and $2E\ \ 3V$. The faces of the polyhedron are polygons, each bounded by a number of sides. Along each edge exactly two faces come together, so an edge corresponds to exactly two sides: the total number of sides is $2E$. We also notice that any face has *at least* 3 sides, so the total number of sides is *at least* 3 times the number of faces. Thus we get:

The total number of sides = $2E$

and

The total number of sides $3F$.

Putting this together we get:

$2E\ \ 3F$,

proving our first inequality.

To prove the second inequality we count the total number of *ends* of edges. Each edge has two ends, so the total number of ends equals $2E$. At each vertex at least three edges come together, so the total number of ends of edges is at least 3 times the number of vertices. Putting this together we get:

2E 3V.

Now, if a polyhedron has 7 edges, then *3F 14* and *3V 14*. This means that both *F* and *V* cannot be bigger than 4. A little thought will convince you that every polyhedron has strictly more than three faces, so we must have *F=4*. Similarly we get that *V=4*. This gives

$V - E + F = 4 - 7 + 4 = 1 \;\; 2.$

This tells us that our hypothetical seven−edged polyhedron cannot exist, for if it did, then Euler's formula would hold and the above sum would have to be equal to 2 QED!

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