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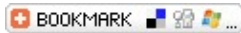
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Features



How maths killed Lehman Brothers

by Horatio Boedihardjo



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On a sunny morning in 2001, a piece of investment plan landed on the desk of Dick Fuld, the then Chief Executive of Lehman Brothers. The document, compiled by a team of maths and physics PhDs, included a calculation to show how the bank will always end up with a profit if they invest on the real estate markets. Fuld was impressed. The next five years saw the bank borrowing billions of dollars to invest in the housing market. It worked. The housing market boom had turned Lehman Brothers from a modest firm into the world's fourth largest investment bank.



The Lehman Brothers headquarters, Rockefeller Center, New York, before the collapse. Image: [David Shankbone](#).

But as the housing market started to shrink, the assumptions that the PhDs made began to break down one by one. The investment now became a mistake, resulting in a stunning loss of \$613 billion. On September 15 2008 Lehman Brothers collapsed "The largest bankruptcy in the US history," as described by Wikipedia.

The money making model

Imagine that you are working for Lehman Brothers and one morning you receive a phone call from HSBC.

"Hi! A hundred customers have each borrowed \$1 million from us for a year. We would like to buy an insurance from you which will cover us in the case of any of them defaulting. From their application forms we reckon they each have a 3% chance of default. How much will the insurance cost?"

You can in fact calculate it, easily. The 100 customers each have a 3% chance of defaulting, so you expect three customers to default next year. That is, you will need to pay \$3 million next year. Assuming the interest rate is about 3% each year, next year's \$3 million would be worth $3/(3/100+1)=3/1.03=2.91$ million now.

Therefore HSBC will have to pay you at least \$2.91 million for the insurance. Obviously Lehman Brothers wasn't a charity and so, to make money, they would double the price to \$5.82 million and expect to make \$2.91 million out of each of these deals on average. This kind of insurance is called a *credit default swap* (CDS).

The legendary CDO

After putting down the phone, you might be quite worried about what would happen if ten of the borrowers defaulted, because then you would have to pay \$10 million back! In this case, consider this deal: how about paying me a certain premium, and if more than ten defaults occur, you will only need to pay for ten of them and I will pay for the rest. If less than ten defaults occur, you will have to pay for all the defaults and I won't pay anything. The type of deal that I am offering is called the *senior tranche* of a *collateralised debt obligation* (CDO) contract, while the one you are getting is called the *junior tranche* of the CDO.



Looks like a safe investment? Better think twice!

The attractiveness of the senior tranche is that almost all of the time I don't have to pay anything, just pocketing my premium. Imagine how unlikely it is to have more than ten borrowers defaulting together! Senior tranches were generally considered to be almost as safe as borrowing money from the government. Since the senior tranche offers a better return than, but seems to be just as safe as, putting the money in the bank, the investors just loved it. In 2004 there were only \$157.4 billion of CDO being issued, but by 2007 the amount grew to \$481.6 billion.

But don't you find it a bit unfair that you can have something as safe as bank deposits, that offers a higher return?

The pitfalls

Yes, it is unfair! In fact, CDO is a lot riskier than bank deposits, but Lehman Brothers, like many investors, didn't seem to know that. Let's go back to our original model. The first source of error is that we have assumed that each investor has a 3% chance of defaulting. How do we know that? It must be from historical data. The problem is, there hasn't been a national drop of housing price since the great depression in the 1920s, so the chance that a borrower defaults was calculated on the basis of a good period when the housing prices surged. However, the housing market crashed in 2007. Many borrowers' properties are now worth even less than the loan they have to pay in the future, so many of them refuse to pay. To worsen the situation, 22% of these borrowers are the so-called *subprime* borrowers those who had little income and had little hope of returning

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money. Banks were not afraid of lending money to them because even if they defaulted, the insurance would pay them back. The participation of the subprime borrowers makes lending much riskier than before.

In fact, the default probability in the US has quadrupled from the 3% as assumed in the model to 12% since 2007, making it four times riskier. This means that investors like Lehman Brothers will be hit four times harder than they have anticipated.

Actually it is worse than that. The profitability, or lack of it, of financial products more complicated than CDS and CDO may depend on the square of the default probability, rather than the probability itself. Now as the default probability rises from 0.03 to 0.12, the square of the probability increased from 0.0009 to 0.0144 that's an increase by a factor of 16!



The Lehman Brothers headquarters on the night of September 15, 2008. Image: [Robert Scoble](#).

There is also a second and more subtle source of error. Whether you can make money from selling the CDO insurance for the bank depends on whether the borrowers return the money, which in turns depends on the economy. So if the economy goes down, you are a lot more likely to lose money. If you are an active investor, then you probably have invested in the stock market as well. Now if the market crashes you lose both the money invested in the stock market and in the CDO. Suppose, on the other hand, that instead of spending the money on CDO, you bet on whether Manchester United will win the European Champion League. This time in order to lose all your money you need both the market to go down *and* Manchester United to lose their match this is less likely than just having the market go down. Therefore, investing on CDO is a riskier choice than betting for Manchester United. The error in our model is that we have not taken into account this extra risk due to its dependence of CDO on the market.

These two errors were sufficient to mask the risk in CDO. In fact, the errors are so serious that 27 out of 30 of the CDOs issued by Merrill Lynch were downgraded from AAA (the safest investment) to "junk" when the errors were spotted.

The fall of Lehman Brothers

Lehman Brothers, unaware of the hidden risks, decided to invest big on CDO. It even had a 35 to 1 debt to equity ratio, that is, it only owned \$1 out of every \$36 in its bank account the other \$35 were borrowed from somewhere. This meant that a loss of just 3% of the money on its balance sheet, would have meant the loss of all the money it owned. After suffering heavy losses (more than 3% of the money in its balance) from CDO, borrowers began to lose confidence and called back the loans. As Lehman had always relied on short-term loans, its lenders were able to pull their cash back quickly. Now the bank was in trouble. It borrowed much more than it was able to return and soon found itself unable to pay back. On 15th September 2008, the world's fourth biggest investment bank was gone, forever.

About the author



Horatio is studying for a doctorate in probability at Oxford University. He enrolled at Oxford University as a maths undergraduate when he was 15 and had to reside with a guardian. After spending four months unsuccessfully searching for one, his dad decided to quit his management job in Hong Kong to accompany his study. Horatio would therefore like to dedicate this article to his loving father for all his sacrifice. He would also like to thank Iris for her encouragement when he almost gave up writing this article.



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