FINDING YOUR WAY AROUND THE ARCTIC

Where am I going?

The aim of the Catlin Arctic Survey is to gather information to better understand the future of the Arctic sea ice cover. It's not just about getting to the North Pole, but about getting there along a route that enables the ice team to gather as much data as possible. The route taken by the ice team has been carefully planned with the help of scientists using a detailed map of the Arctic – but how do you make such a map?

"Good navigation tools are essential – our life depends on them!"

Pen Hadow
Expedition Leader

photograph © Martin Hartley www.martinhartley.com

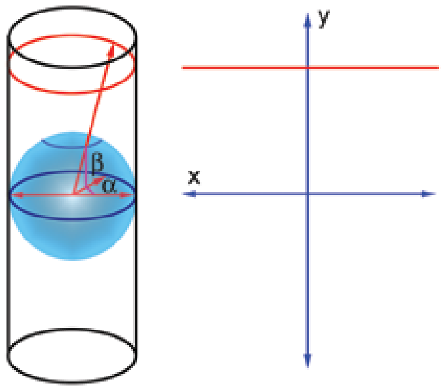
The route taken by the ice team has been carefully planned

A map is a way of picturing the spherical Earth on a flat piece of paper. Over small local areas, the surface of the Earth is very similar to a flat sheet, and so it is easy to accurately draw the geographical features of that area onto a piece of paper, keeping distances accurate.

But what if you are interested in mapping a much larger area – the ice team are covering a very long distance – or perhaps even the whole globe itself? If you imagine wrapping a sheet of paper around an orange, at some point you will have to fold or scrunch up the paper in order to make it fit around the spherical orange. Any two-dimensional map of the whole globe is not going to accurately represent the spherical Earth. So just how do you map the sphere onto the 2D page?

The Mercator projection

Map-makers use what is called a *projection* to depict a three-dimensional object (such as the globe of the Earth) in two dimensions. One of the most famous maps of the world is called Mercator's projection. Invented in the 16th century by the Flemish cartographer Gerardus Mercator, this map is a projection of the globe onto a cylinder wrapped around the equator, which is then cut along one side to create a two-dimensional map.



The Mercator projection projects the Earth on a cylinder and then cuts it open

The simplest way to project the surface of the sphere onto the cylinder is to draw a line from the centre of the Earth through each point on the surface, and mark where that line intersects the cylinder. Then each point on the Earth's surface (given by its longitude α and its latitude β) is mapped to a point (x,y) on the 2D map with coordinates

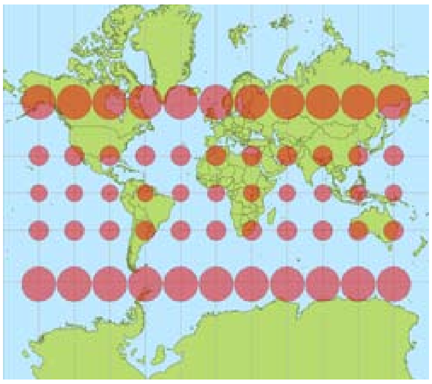
$$x = \alpha$$
$$y = R \tan(\beta)$$

Here R is some constant which scales the map so it can fit on your page. If we assume R=1, then London, with longitude and latitude approximately (0,51), maps to the point with coordinates (0, 1.23), St John's on the coast of Newfoundland, Canada, with longitude and latitude approximately (52, 47), maps to (52, 1.07). But as you get closer and closer to the North and South Poles, the distances get stretched, until the Poles actually disappear off the map altogether (tan (90) and tan(-90) are not defined).

Mercator modified this projection, by taking the y coordinate in his map to be

$$y=R\ \ln(\tan(\beta/2+45))$$

This has the effect of reducing the distortion of the distances close to the Poles, but distance and area far from the equator are still exaggerated (eg. Greenland appears to be larger than Africa, when in reality Africa is over 14 times bigger).



Images ©Stefan Kühn

The distortion of the equally sized circles shows how the Mercator Projection distorts area

Although Mercator's projection comes with distortion, it does have one important feature that means it is still in use for navigation today – it maps lines of constant bearing on the globe to straight lines on the map. This makes Mercator's map very useful for sailors and nautical navigation – to sail from London to Newfoundland, they measure the bearing by drawing a straight line connecting the two places on Mercator's map and set sail in the direction indicated by the orientation of the straight line.

But there are also other projections. The Gall-Peters projection, for example, preserves area, but greatly distorts shapes. Some people prefer this map for political reasons: in Mercator's projection Europe appears much larger and almost at the centre of the map, while the Gall-Peters projection depicts relative sizes accurately.

Image ©NASA.

The Gall-Peters projection

The important point to realise is that no one projection is correct

In fact there are many projections used to map the world. The one we see in most atlases and world maps is the *Winkel tripel* projection – used by National Geographic for world maps, in the Times Atlas, and many school textbooks. This projection is more complex – it averages the coordinates from two other projections, the *Aitoff* projection and the *equirectangular* projection. The Winkel tripel projection doesn't preserve area or distance, but gives a balance with no one type of distortion overweighing.

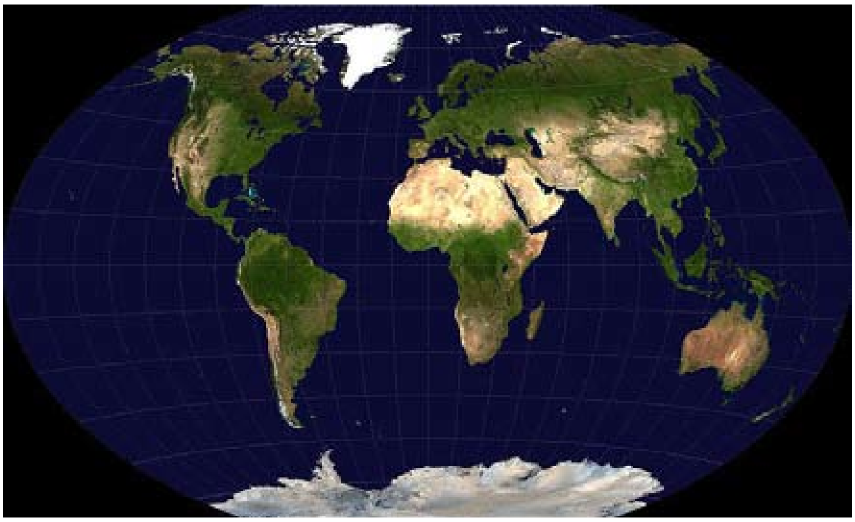


Image ©NASA.

The Winkel tripel projection

The important point to realise is that no one projection is correct – each has a distortion and you will never be able to create a map that accurately shows area, size, distance and direction for the whole globe at once. Instead you have to pick the best map for the job.

So what map should our intrepid explorers, Pen, Ann and Martin, use to plot their progress to the North Pole? The projections above are no use as they don't even show the Poles. One option might be to use a *transverse Mercator* projection where the cylinder wraps around the Earth on a line of longitude running through the Poles rather than around the equator.

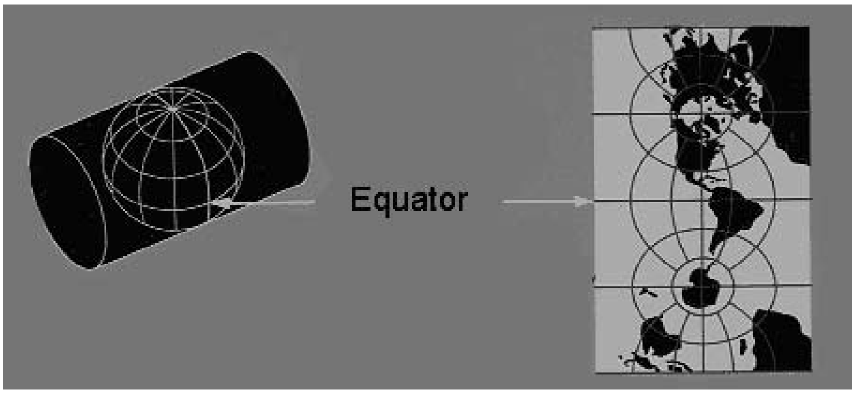
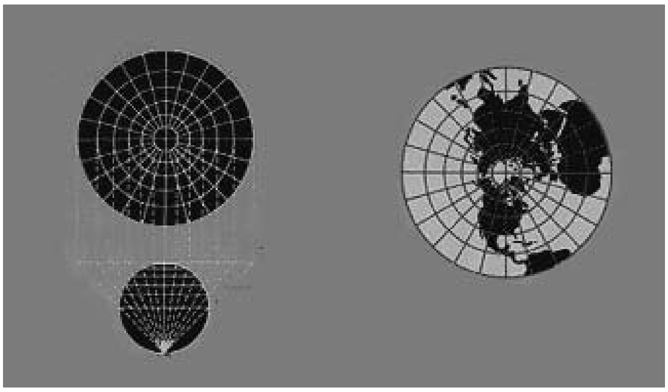


Image © United States Geological Survey.

The transverse Mercator projection

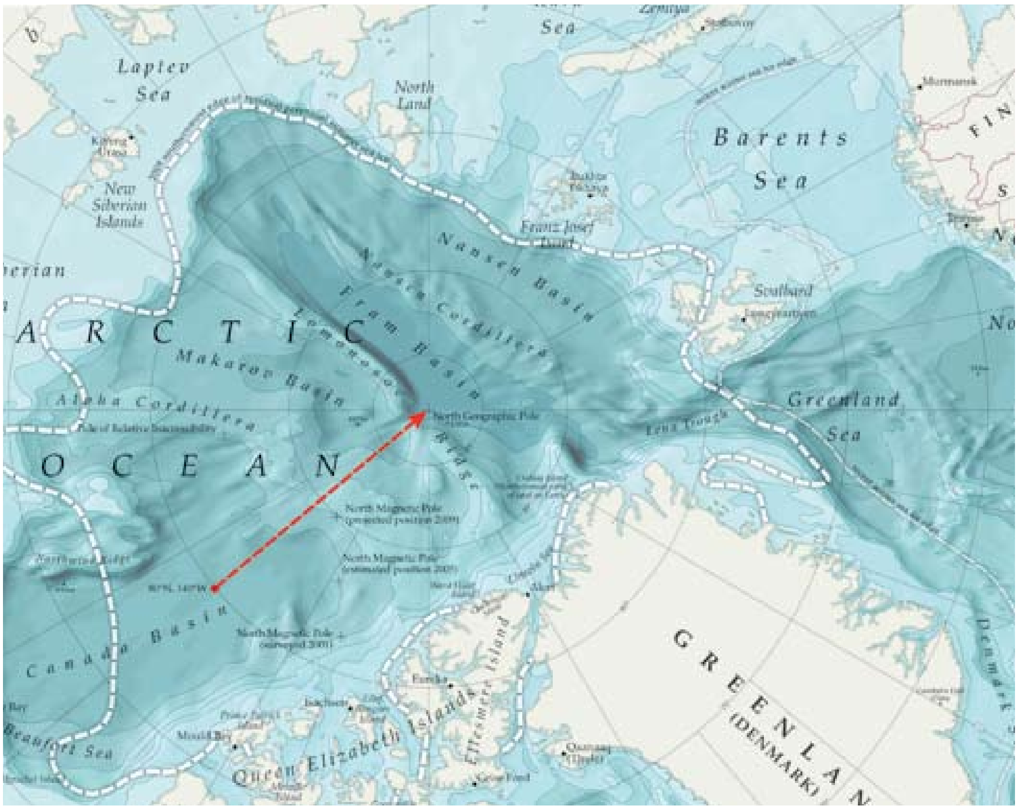
Imagine balancing a stiff sheet of paper on top of the world

But perhaps the best choice would be a stereographic projection, like that used to produce the map showing the expedition route. This is a fairly simple projection: imagine balancing a stiff sheet of paper on top of the world, touching at the North Pole and projecting points from the South Pole onto the paper.



The stereographic projection

This projection preserves angles. We can see the lines of latitude are concentric circles around the Pole in the centre of the picture and the lines of longitude are straight lines radiating out from the North Pole. Here is the map showing the explorers' intended route:



The map shows the intended route of the ice team

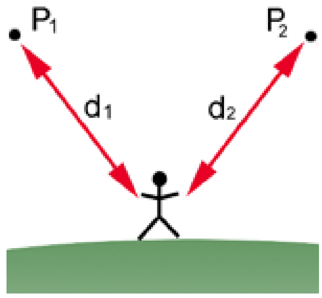
"Without GPS technology, the ice team would have to navigate by the stars!"

Simon Harris-Ward
Operations Director

But where am I?

It's all very well having the right map, but if you don't know where you're standing, how can you know where you're going? There are few landmarks on the Arctic to navigate by – one iceberg looks much like another – and even the stars may not be visible in bad weather. Luckily, the ice team are equipped with GPS (the global positioning system). The GPS satellites that orbit the Earth continually transmit messages containing the exact time of transmission, and the exact location of the satellite at the time of transmission. When the GPS receiver receives these messages, it uses the time delay to work out exactly how far away it is from the satellite.

In the plane two points of reference are sufficient to work out your location: if you know that you are at distance d_1 from point P_1 and at distance d_2 from point P_2, then you know that you are somewhere on the circle around P_1 with radius d_1 and somewhere on the circle around P_2 with radius d_2. These two circles meet in at most two points, so you know that you are at one of these two points, and some additional information will help you to work out exactly which of the two it is.



In three dimensions a similar rule applies, although here you need three points of reference. If you know that you are at distance d_1 from point P_1, distance d_2 from point P_2 and distance d_3 from point P_3, then you know that you must be on the sphere around P_1 with radius d_1, and on the sphere around P_2 with radius d_2, and on the sphere around P_3 with radius d_3. In general, two intersecting spheres intersect in a circle. A third sphere intersects that circle in at most two points. So again, with a little additional information, you can work out where you are once you know your distance from three points.

And this is exactly how GPS works: the GPS receiver uses the signals from at least three satellites to work out its distance from each, and then to work out its exact location by calculating the intersections of spheres. In fact, to minimise error and to gain additional information, GPS receivers use four satellites.

photograph © Martin Hartley www.martinhartley.com

Ann checking her GPS position before setting off in the morning

Further reading

- Thomas Harriott: a lost pioneer
  http://plus.maths.org/issue50/features/faherty/
- Time and motion
  http://plus.maths.org/issue7/features/greatcircles/
- The fundamentals of mapping
  http://www.icsm.gov.au/mapping/about_projections.html
- Map projections
  http://erg.usgs.gov/isb/pubs/MapProjections/projections.html
- Spherical projections
  http://local.wasp.uwa.edu.au/~pbourke/geometry/spherical